

Prestack elastic waveform inversion using a priori information

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Summary

In order to provide a user guided and quantitative approach to AVO integrated processing, we propose a method which take into account a priori information in prestack seismic data inversions. The approach is based on a formalism in which the a priori information is incorporated in an a priori model of elastic parameters — density, P and S-impedances — and an exponential model covariance operator. Then, a global objective function is minimized to produce an optimal model parameter estimation. We focus on a 2D synthetic example to better understand the performance of this prestack stratigraphic inversion method, especially for improving the S-impedance estimation from PP noisy data.

Introduction

It is well known that seismic inversion cannot reliably estimate all the parameters of the subsurface. Singular value decomposition analysis of the linearized problem allows us to investigate the information contained in seismic data for different acquisition geometries, reflection types, offset ranges and noise level (De Nicolao et al., 1993; Lebrun et al., 1998). Concerning PP data, these authors showed that the P-impedance variation is the best determined parameter. However, the confidence in the estimation of a second parameter from AVO data is very limited, depending on the noise level and the available offset range.

To improve the problem of non-uniqueness and to provide robust model parameter results, it is necessary to use additional available information, such as well log data and geometrical geological knowledge. Using this a priori information, the inversion process now consists in computing the medium parameters consistent with all the known information. A. Tarantola (1987) assumes that the probability density functions describing the data errors and the model parameter uncertainties are Gaussian. Several studies have been carried out following this approach: for example for poststack data (Brac et al., 1988) and more recently for prestack data (Pan et al., 1994; Simmons et al., 1996). We also adopt these assumptions and propose a “geological” objective function which offers many possibilities to incorporate a priori information.

In the following, we first describe the method for constraining prestack inversion with available geological information. Then, the method is illustrated on a 2D synthetic example corresponding to marine acquisition and to a realistic reservoir zone. Linearized inversion of noisy prestack PP reflection data is performed, both without and with a priori information. The results are analyzed

and compared to the exact reservoir parameters.

Prestack stratigraphic inversion method

We adopt a Bayesian inverse calculation to estimate elastic impedances from seismic data as thoroughly developed by A. Tarantola (1987). In practice, if the seismic noise is described by a Gaussian probability with zero mathematical expectation and covariance operator C_d , and if the uncertainties on the model are described by a Gaussian probability with zero mathematical expectation and covariance operator C_m , then the maximum likelihood model minimizes the sum of two objective functions:

$$J = J_s + J_g,$$

where J_s and J_g are respectively the seismic and “geological” objective functions. J_s measures the mean square error between model-predicted and actual prestack data. We assume that the seismic noise is uncorrelated from one trace to another: the data covariance C_d is diagonal, with a seismic variance σ_s^2 function of the average noise level in the seismic data. J_g measures the error between a priori (m^{pr}) and predicted model parameters:

$$J_g = \int \int (m - m^{pr})(\mathbf{x})(C_m^{-1})(\mathbf{x}, \mathbf{x}')(m - m^{pr})(\mathbf{x}')d\mathbf{x}d\mathbf{x}',$$

where $m(\mathbf{x}) = (m_1(\mathbf{x}), m_2(\mathbf{x}), m_3(\mathbf{x}))$ represents the vector of model parameters and $(C_m^{-1})(\mathbf{x}, \mathbf{x}')$ is the kernel of the inverse of the covariance operator. The choice of C_m , fundamental in our approach, is derived from the one used in the INTERWELL software for the 2D poststack stratigraphic inversion option (Brac et al., 1988). We use correlation lines derived from interpreted horizons and stratigraphic knowledge, and well logs in the depth domain, to fill the inter-well volume using a standard interpolation technique. Consequently, we obtain a priori elastic model in depth that needs to be refined because it doesn't explain the seismic amplitude variation both in space and with offset. Concerning the kernel C_m , a priori information is incorporated by means of user defined additional parameters: a variance $\sigma_i(\mathbf{x})$ on $m_i(\mathbf{x}) - m_i^{pr}(\mathbf{x})$, a correlation coefficient $\rho_{ij}(\mathbf{x})$ between $m_i(\mathbf{x}) - m_i^{pr}(\mathbf{x})$ and $m_j(\mathbf{x}) - m_j^{pr}(\mathbf{x})$, and a correlation length λ . Finally, we assume that the covariance is exponential along the correlation lines and diagonal in the orthogonal direction. Then, in the coordinates system in which s is the length along the correlation lines and τ is the length in the orthogonal direction, the kernel of the covariance operator is:

$$C_m(s, \tau; s', \tau') = \sigma(s, \tau; s', \tau') \exp\left(-\frac{|s - s'|}{\lambda}\right) \delta(\tau - \tau'),$$

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where

$$\sigma(s, \tau; s', \tau') = P(s, \tau)D^{1/2}(s, \tau)D^{1/2}(s', \tau')P^T(s', \tau'),$$

with P and D , respectively the orthonormal and diagonal matrices defined by the decomposition of the real positive symmetric matrix $\sigma(s, \tau; s, \tau)$:

$$\begin{aligned} \sigma(s, \tau; s, \tau) &= \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 \\ \rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix} (s, \tau) \\ &= (PDP^T)(s, \tau) \end{aligned}$$

This formalism allows specific variations of the variance σ_i and of the correlation coefficient ρ_{ij} along the correlation lines. C_m^{-1} can then be computed in the original variables x, z , and as in the constant σ case (Tarantola, 1987), this operator is differential (its matrix is sparse).

In the present seismic objective function, we have used a linearized forward modeling tool to generate the synthetic CDP gather. This modeling which assumes 3D propagation and 1D model perturbation is derived from Lebrun et al. (1998). Minimization of the objective function is performed by a conjugate gradient technique.

Synthetic study

Our marine synthetic example is designed to illustrate the performance of prestack stratigraphic inversion. The model contains a 2D target zone, derived from the Mesa Verde outcrop (Colorado, USA) and embedded in a 1D background. For a more detailed description of this model see Bourgeois et al. (1994). Both P and S-impedance models can be seen in Fig. (1a) and (1e). The propagation velocity in the background medium is independent of the horizontal position. Consequently, the synthetic data were efficiently calculated using the linearized modeling technique described by Lebrun et al. (1998). In the target window, our synthetic CDP gathers each contains 50 offsets traces with P-wave primary reflections only. Random noise with an 80% noise to signal ratio was also added (Fig. (3a)).

We first run the inversion without using a priori information. The P and S-impedance results are shown in Fig. (1c) and (1g), and compared with the exact filtered impedances of Fig. (1a) and (1e). As expected, the P-impedance is well recovered although slightly contaminated by noise in the data. The S-impedance result is quite poor, as mentioned in De Nicolao et al. (1993) and Lebrun et al. (1998).

The a priori model was created using P and S-impedance logs available at two opposite locations ($x=250\text{m}$ and $x=5750\text{m}$), three depth horizons interpreted from the inversion results without using a priori information and, finally, a stratigraphic framework specified as conformable with the bounding horizons. The a priori P and S-impedance models are shown in Fig. (1b) and (1f).

Constraints were set within each of the four defined geological units to limit the range of solutions to those which

were geologically admissible. We selected the covariance operator parameters according to some information about the lateral heterogeneity of the model parameters within each unit. During all the inversion experiments, a single scalar was applied for each standard deviation: $\sigma_s = 80\%$, $\sigma_{I_p} = 5\%$ and $\sigma_{I_s} = 10\%$. We also assumed that the elastic parameters uncertainties are independent ($\rho_{ij} = 0$).

In a first constrained prestack inversion, we assumed that the impedance values were strongly correlated within units U_1 and U_4 ($\lambda_{U_1} = \lambda_{U_4} = 500\text{m}$), whereas they were highly varying laterally within units U_2 and U_3 ($\lambda_{U_2} = \lambda_{U_3} = 50\text{m}$). The final inversion results are shown in Fig. (1d) and (1h). Thanks to this inversion formalism and to selection of a priori information “consistent” with the actual model, only relevant information is integrated in the model. The a priori information provided a much more detailed picture of the target zone. Of particular interest is the impedance feature in unit U_3 where the lateral continuity has been improved, even with a small correlation length. As expected, by using a priori information in prestack inversion, the updates for P-impedances were rather small compared to those of S-impedances. In addition, S-impedance results indicate that reliable and quantitative results have been delivered by inversion. The final seismic residuals (Fig. (3b) and (4b)) mainly contains the incoherent noise that was added to the data.

In a second run we assumed a different scenario: laterally heterogeneous impedances in units U_1 and U_2 ($\lambda_{U_1} = \lambda_{U_2} = 50\text{m}$) and more continuous impedances in units U_3 and U_4 ($\lambda_{U_3} = \lambda_{U_4} = 3000\text{m}$). The inversion results (Fig. 2) clearly illustrate that the rate of lateral variation in the final model is controlled by the correlation length λ . As some of the model covariance parameters are wrong within this run, the inversion produces impedance results which are not compatible with all the available information, especially in the data space (Fig. (3c) and (3d)).

Conclusions

The general form of the model covariance matrix proposed for poststack inversion has been extended for a multi-parameter inverse problem, with a first application for prestack stratigraphic inversion. The two-fold objective function allows to balance the confidence between seismic data and geological interpretation. It allows to incorporate the amount of geology and seismic corresponding to coherent information into optimal P and S-impedance models. These models represent estimates of the absolute values that can ease a petrophysical interpretation. This truly integrated prestack inversion is a flexible approach that could be easily extended to 3D with respect to the model covariance matrix and to other seismic forward modeling tools for the seismic objective function.

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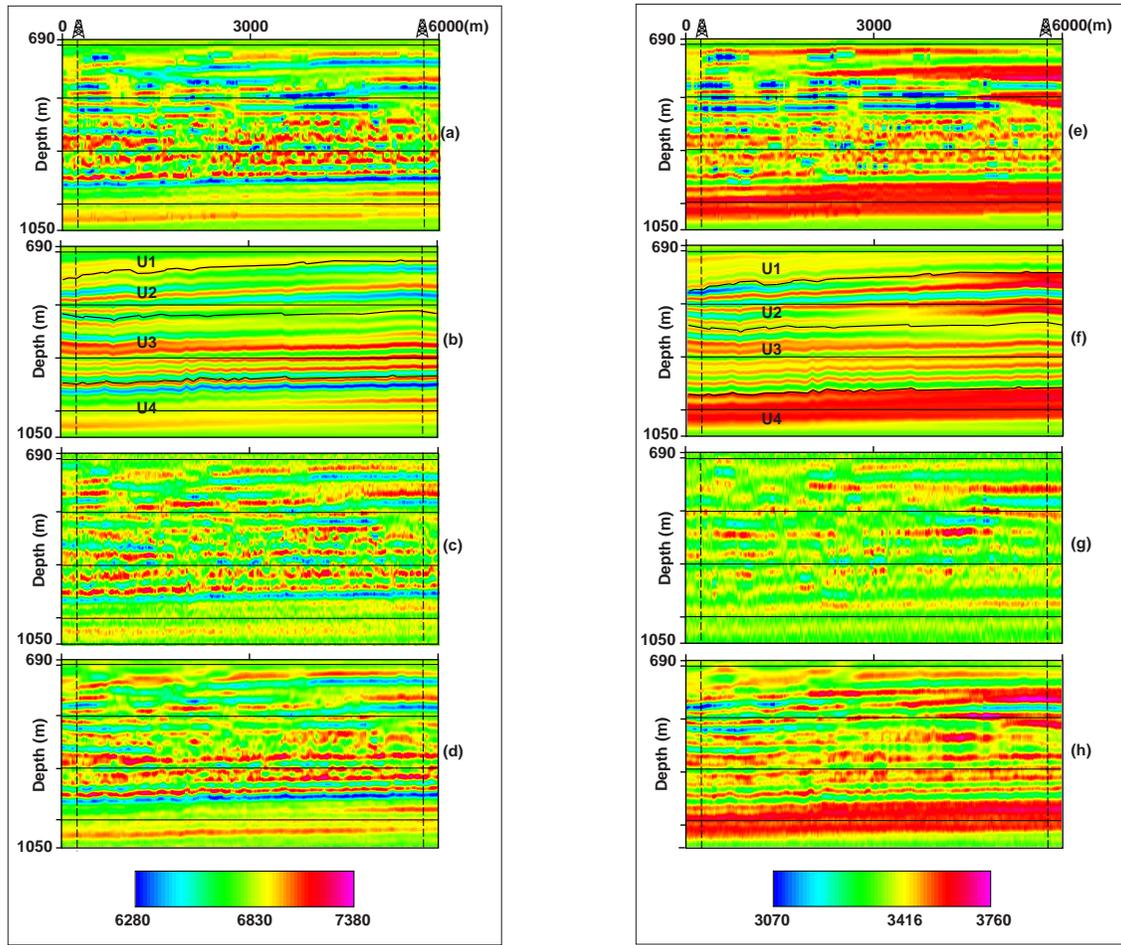


Fig. 1: P (left) and S (right) impedance (a) and (e) exact models after vertical filtering - (b) and (f) a priori models - (c) and (g) inversion results obtained without a priori information - (d) and (h) inversion results obtained with realistic a priori information

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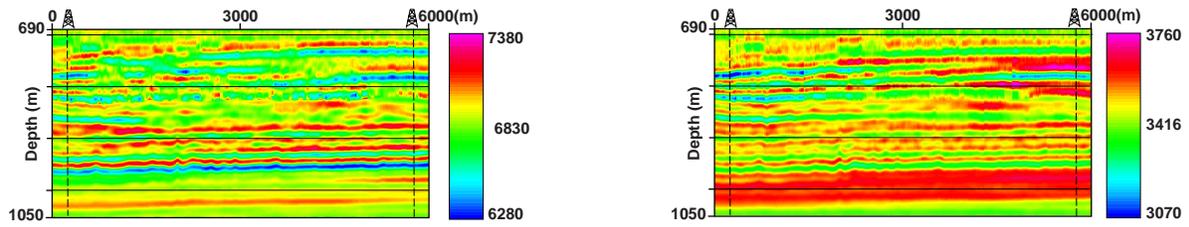


Fig. 2 P (left) and S (right) impedance results obtained with a wrong a priori information ($\lambda_{U1}=\lambda_{U2}=50$ m, $\lambda_{U3}=\lambda_{U4}=3000$ m).

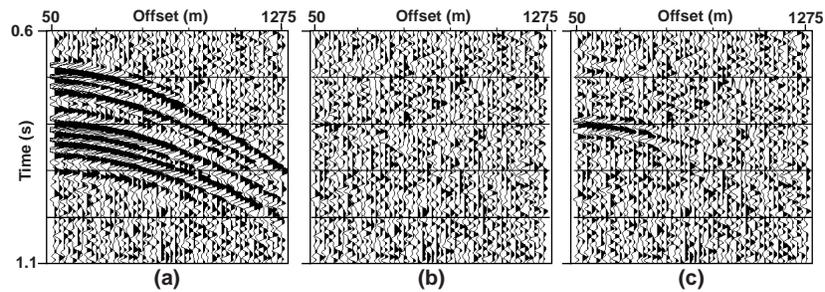


Fig. 3 CDP gather at location 3000 m - (a) observed - (b) residuals: inversion with realistic a priori information - (c) residuals: inversion with wrong a priori information.

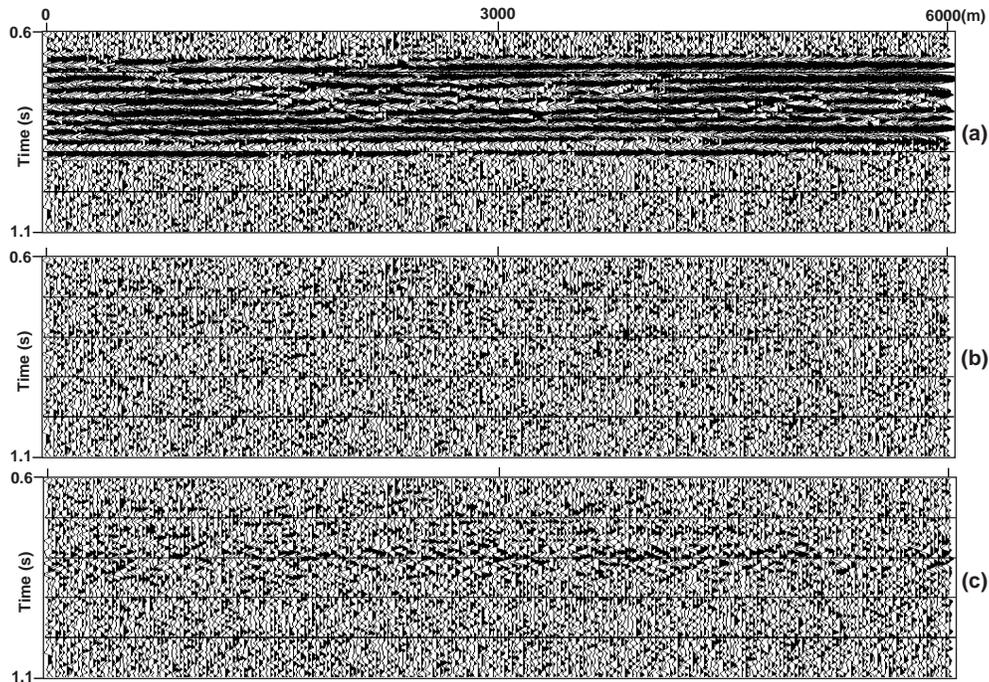


Fig. 4 Common offset gather at offset 50 m - (a) observed - (b) residuals: inversion with realistic a priori information - (d) residuals: inversion with wrong a priori information.