

A New Macroscopic Model for One-Phase Miscible Flow in Heterogeneous Media

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Abstract

Presently, the most attractive empirical models for representing macroscopically viscous fingering behavior in miscible displacements are from Fayers, Jouaux, and Tchelepi and from Todd and Longstaff named hereafter respectively FJT model and T&L model. Although successful in representing most of the cases investigated in their paper, the FJT formulation requires two phases and two components to represent the behavior of a single non ideally mixed phase with two components.

The transition between a two-phase multi-component model to a two-phase two-component model, as a Fayers type model is presented. Then, a new fingering function is introduced, which allows for getting a single phase two-component model for representing fully miscible displacements at unfavorable viscosity ratios.

The new model, - which is shown to reproduce very closely the pressure drop and fractional flow obtained by Fayers et al. with their so-called ' Λ_H ' function for any mobility ratio, heterogeneity factor or gravity number -, demonstrates the dependence of the effective viscosity of the single 'non homogeneous' phase on the heterogeneity, the effective phase behavior appearing then not to be purely of thermodynamic type.

Introduction

The instabilities occurring in first-contact miscible displacements at unfavorable viscosity ratio lead to development of viscous fingers and decrease the sweep efficiency. From a thermodynamic point of view, only one phase is present in the porous medium since there is no interfacial tension between the injected and displaced fluids. The proper simulation of the single phase flow at a macroscopic scale requires an appropriate scaling-up of the component material balance equations together with an appropriate thermodynamic model for the calculation of the physical properties (viscosity, density) of the unique phase ; this latter requirement is far more unexplored than the former.

For gas-oil systems, a compositional tool, as a multi-component finite difference compositional model (one of the earliest being the one proposed by Coats ¹), would be used to simulate the composition of the unique phase. But the single phase physical properties are usually still obtained with the assumption of a full mixing of the components at the scale of the grid block, not relevant at a macroscopic scale.

The first empirical models, Koval ² and Todd and Longstaff ³, for representing macroscopically viscous fingering behavior in fully miscible displacements, have been obtained from a ‘Black-Oil’ approach : each component, considered as a phase, has an effective viscosity and density. The chief contribution of Koval is the introduction of a component fractional flux function based on an effective viscosity ratio, and a heterogeneity factor, shown ⁶ to reproduce quantitatively the Blackwell *et al.* experimental results⁷.

More recently, Fayers^{4,5} have used a two-component two-phase approach with physically interpretable parameters, based on a so-called fingering function corresponding to a relationship between the sub-phase gas saturation and its composition. Then Fayers, Jouaux, and Tchelepi ⁶ have introduced a new fingering function including the heterogeneity factor from Koval.

The FJT model performance has been found superior to that of the T&L model ⁶. The draw-back of the formulation is that it requires two phases and two components to represent the behavior of a single non ideally mixed phase with two components.

The present paper examines first the transition from a two-phase multi-component model to a two-phase two-component model, as a Fayers type model. Then, a new fingering function is introduced, which allows for getting a single phase two-component model for representing fully miscible displacements at unfavorable viscosity ratios, and also an extension of the Koval’s model.

The new models reproduce very closely the pressure drop and fractional flow obtained by Fayers et al. with their so-called ‘ Λ_H ’ function for any mobility ratio, heterogeneity factor or gravity number. The effective phase properties of the single ‘non homogeneous’ phase are shown to depend on the heterogeneity factor, the effective phase behavior appearing then not to be of pure thermodynamic type.

Unidirectional two-phase multi-component displacement

The k-component material balance equation of such a problem, ignoring source and diffusion/dispersion terms, are usually written in a compositional simulator as :

$$1. \quad \partial_t [\phi (\xi_o S_o x_k + \xi_g S_g y_k)] + \partial_x (\xi_o u_o x_k + \xi_g u_g y_k) = 0 .$$

A full description of the symbols is given in the nomenclature. The component molar fractions are named x in the oil phase, y in the gas phase. The equilibrium constants, and the phase physical properties are obtained from a PVT model.

The closure equations of the problem are :

$$2. \quad S_o + S_g = 1 ; \sum x_k = 1 ; \sum y_k = 1 .$$

The phase velocities, from the Darcy law, can be written as :

$$3. \quad u_g = u_t \left(\frac{\lambda_o}{\lambda_t} - \frac{\lambda_o \lambda_g k \Delta \rho g \sin \varphi}{\lambda_t u_t} \right) \quad u_o = u_t \left(\frac{\lambda_o}{\lambda_t} + \frac{\lambda_o \lambda_g k \Delta \rho g \sin \varphi}{\lambda_t u_t} \right)$$

where λ_p (p = o,g) is the phase mobility : $\lambda_p = \frac{k r_p}{\mu_p}$, λ_t is the total mobility : $\lambda_t = \lambda_o + \lambda_g$. $\Delta \rho$ is the difference between the phase densities : $\Delta \rho = \rho_o - \rho_g$. φ is the angle between the horizontal and flow directions.

The total velocity u_t , defined by : $u_t = u_g + u_o$, is given by :

$$4. \quad u_t = -k \lambda_t \left[\partial_x P - \left(\frac{\lambda_g}{\lambda_t} \rho_g + \frac{\lambda_o}{\lambda_t} \rho_o \right) g \sin \varphi \right].$$

Towards a two-phase two-component model

Applying this problem to the reduction to one dimension of a fully-miscible unstable displacement in a cross-section, the oil saturation variable can be considered as representing the fractional volume of not-contacted oil bypassed by the fingers ; the bypassed oil keeps its initial composition described by the fractions x_k^i . Furthermore, if all the injected components are assumed to keep the same velocity and dispersivity, the composition of the gas sub-phase is given, at any time and position, by :

$$5. \quad y_k = L x_k^i + V y_k^i = x_k^i + V (y_k^i - x_k^i),$$

where the molar fractions y_k^i describe the composition of the injected fluid, L and V being the molar fraction of respectively initial oil and injected gas in the gas sub-phase ($L + V = 1$).

The gas phase density is then given by :

$$6. \quad \rho_g = \xi_g \sum_k MM_k (L x_k^i + V y_k^i) = \xi_g (MM_L L + MM_V V) = \xi_g \left(\frac{\rho_L}{\xi_L} L + \frac{\rho_V}{\xi_V} V \right),$$

MM_k is the k-component molar mass, MM_L and MM_V are respectively those of the oil and of the injected gas (both of constant composition), ρ_L and ρ_V their phase densities, ξ_L and ξ_V their phase molar densities.

Summing the conservation equations 1 for all the components leads to :

$$7. \quad \partial_t (\phi \xi_r) + \partial_x (u_i \xi_r) = 0,$$

with $\xi_r = \xi_L S_o + \xi_g S_g$, $\xi_f = \xi_L G_o + \xi_g G_g$, and where G_p ($p = o, g$) is the phase fractional flux : $G_p = \frac{u_p}{u_t}$.

ξ_r represents the resident non-homogeneous hydrocarbon phase molar density, and ξ_f represents the flowing non-homogeneous hydrocarbon phase molar density.

Given the equations 5 and 7, the k-component conservation equation becomes :

$$8. \quad (y_k^i - x_k^i) [\partial_t (\phi \xi_g S_g V) + \partial_x (\xi_g u_g V)] = 0.$$

The resolution of the two-phase multi-component problem is hence analogous to that of a two-phase two-component problem (L and V) such that :

$$9. \quad x_v^i = 0 ; x_L^i = 1 ; y_v^i = 1 ; y_L^i = 0 \Rightarrow y_v \equiv V.$$

All the material balance equations reduce then to the following equations :

$$10. \quad \partial_t (\phi \xi_g S_g y_v) + \partial_x (\xi_g u_t G_g y_v) = 0$$

$$11. \quad \partial_t [\phi (\xi_L S_o + \xi_g S_g y_L)] + \partial_x [u_t (\xi_L G_o + \xi_g G_g y_L)] = 0$$

The average molar fraction of injected gas in the resident hydrocarbon phase, z_v is given by :

$$12. \quad z_v = \frac{\xi_g Z_v}{\xi_L S_o + \xi_g S_g} \quad \text{with} \quad z_v = S_g y_v,$$

and the corresponding average molar fractional flux is given by :

$$13. \quad Z_v = \frac{\xi_g G_g y_v}{\xi_L G_o + \xi_g G_g} \quad \text{with} \quad Z_v = G_g y_v.$$

Usually, in a miscible gas-oil case at high viscosity ratio, the phase molar densities are different. It is to be noted that, to get the context in which most of the miscible models²⁻⁶ have been developed, ξ_g , ξ_L and ξ_V have to be assumed equal : as a matter of fact, this assumption is equivalent to assuming the gas phase density to be given by :

$$14. \quad \rho_g = \rho_v y_v + \rho_L (1 - y_v).$$

With this assumption, there is an identity between z_v and z_o , between Z_v and Z_o , between ξ_r and ξ_f .

Another assumption usually met in miscibility models, the fluid incompressibility, may be not necessary, as far as ρ_g , ρ_v and ρ_l are calculated at each current pressure. This remark holds also for phase molar densities and phase viscosities.

From equation 3, the gas-phase fractional flux G_g can be written as :

$$15. \quad G_g = F_g (1 - \Gamma) , \quad \text{with} \quad \Gamma = N_g \frac{m_v - 1}{m_v} \Delta \rho_n \text{ kr}_o \quad \text{being a gravity correction factor,}$$

$$\text{where } N_g = \frac{k (\rho_l - \rho_v) g \sin \phi}{(\mu_l - \mu_v) u_t} , \Delta \rho_n = \frac{\Delta \rho}{\rho_l - \rho_v} , m_v = \frac{\mu_l}{\mu_v} , F_g = \frac{\lambda_g}{\lambda_l} ,$$

represents respectively the gravity number, the reduced density difference, the viscosity ratio between the oil and the injected gas, the gas phase fractional flux in absence of gravity effect.

Besides, it is also necessary to set a relationship between one of the saturation variables, chosen to be S_g , and one of the composition variables, chosen to be y_v . The key contribution of Fayers⁷ is the introduction of the fingering function concept to express S_g as a function of y_v : at any position in a cross-section, S_g corresponds to the finite fractional width Λ occupied by fingers ($S_g = \Lambda(y_v)$); at the leading edge, the fingers have simultaneously a small fractional width and an oil-rich character (y_v is small), and the fractional width gradually broadens towards the rear with increasing y_v .

One-phase two-component model

If the fingering function is such that both S_g and y_v can be expressed analytically as functions de \mathbf{z}_v , then it becomes possible to write the equations 10 and 11 as :

$$16. \quad \partial_t (\phi \xi_r z_v) + \partial_x (\xi_r u_t Z_v) = 0 , \quad \partial_t (\phi \xi_l z_l) + \partial_x (\xi_l u_t Z_l) = 0 \quad (z_v + z_l = 1, Z_v + Z_l = 1),$$

or, with the assumption of no volume change on mixing :

$$17. \quad \partial_t (\phi \xi z_v) + \partial_x (\xi u_t Z_v) = 0 , \quad \partial_t (\phi \xi z_l) + \partial_x (\xi u_t Z_l) = 0 \quad (z_v + z_l = 1, Z_v + Z_l = 1),$$

then \mathbf{z}_v (z_v) can then be used as a primary unknown in these fully compositional equations of the problem.

Keeping in mind that the oil and gas phases are in fact sub-phases of a single non-ideally mixed hydrocarbon phase, the Darcy velocity of which should be written as :

$$18. \quad u_t = - \frac{k}{\hat{\mu}} [\partial_x P - \hat{\rho} g \sin \phi] ,$$

it follows, per comparison with eq. 4, that the effective flowing hydrocarbon phase properties in the Darcy equation are :

$$19. \quad \hat{\mu} = \frac{1}{\lambda_t} \quad \hat{\rho} = \frac{\lambda_g}{\lambda_t} \rho_g + \frac{\lambda_o}{\lambda_t} \rho_o .$$

It should be noted that, even assuming equal phase molar densities ($\xi_o = \xi_g = \xi$, then $\xi_r = \xi_l = \xi$), the average phase density in the accumulation is not equal to $\hat{\rho}$, but given by the following relationship :

$$20. \quad \rho = \xi (MM_v z_v + MM_l z_l) .$$

Fingering functions

Fayers⁵ has proposed : $\Lambda_1(y_v) = a + b y_v^\alpha$, with $a + b = 1$, for an homogeneous problem, but $a + b \leq 1$ for a heterogeneous case, depending on Peclet number.

Later, Fayers, Jouaux and Tchelepi⁶ have introduced the following fingering functions :

$$\Lambda_H(y_v) = \frac{H m_k - H m_g}{H m_k (1 - H m_g) + y_v H m_g (H m_k - 1)} \quad \Lambda_3(y_v) = y_v^{\beta/(1-\beta)} .$$

The Λ_H function has been developed to fit exactly, in absence of gravity and compressibility effect, the molar fractional flux function of Koval :

$$21. \quad F_k = \frac{m_k H z_v}{m_k H z_v + 1 - z_v} .$$

H is the heterogeneity parameter ($H = 1$ for a homogeneous medium). m_g , the gas-oil viscosity ratio is obtained from a quarter-power blending rule :

$$22. \quad m_g = \frac{\mu_k}{\mu_g} = [y_v (m_v^{1/4} - 1) + 1]^4 .$$

m_k is a constant effective viscosity ratio obtained from equation 22 for a given concentration y_k ($= 0.22$ in the original work of Koval). Considering y_k as the gas concentration just inside the leading edge of the fingers, Fayers *et al.* have proposed :

$$23. \quad \text{If } N_G < 0 : y_k = 0.22 + (0.075 + 0.0018 m_v) \text{ArcTan}(-1.6 N_G), \text{ else : } y_k = 0.22 (1 - N_G^{1.5}).$$

In the FJT model, the relative permeabilities are defined by :

$$24. \quad kr_g = \frac{H S_g}{H S_g + 1 - S_g} \quad \text{and} \quad kr_o = 1 - kr_g .$$

The new fingering function, which is proposed hereafter, is derived from the hypothesis that the V-component molar fractional flux could be a function of the reduced compositional variable $\frac{y_v - y_k}{1 - y_k}$.

Under the same hypotheses as Fayers and using the same relative permeabilities as Fayers *et al.*, if it is assumed that : $F_v = F_g y_v = \frac{y_v - y_k}{1 - y_k}$, then the fingering function is $\Lambda(y_v) = \frac{y_v - y_k}{m_g H y_k (1 - y_v) + y_v - y_k}$.

Providing a modification of the definition of y_k , this fingering function gives results similar to those obtained with the Λ_H function. However, with this function as with Λ_H , it is not possible to derive analytically y_v and S_g from z_v .

In absence of gravity, F_v is expected to behave as F_k . Hence, a more tractable fingering function, allowing for choosing z_v as a primary unknown in a one-phase two-component model, is obtained from keeping the definition of y_k (eq. 23) and stating :

$$25. \quad F_k = \frac{y_v - y_k}{1 - y_k} \quad \text{or} \quad y_v = (1 - y_k) F_k + y_k \quad \text{or} \quad z_v = \frac{y_v - y_k}{m_k H (1 - y_v) + y_v - y_k} .$$

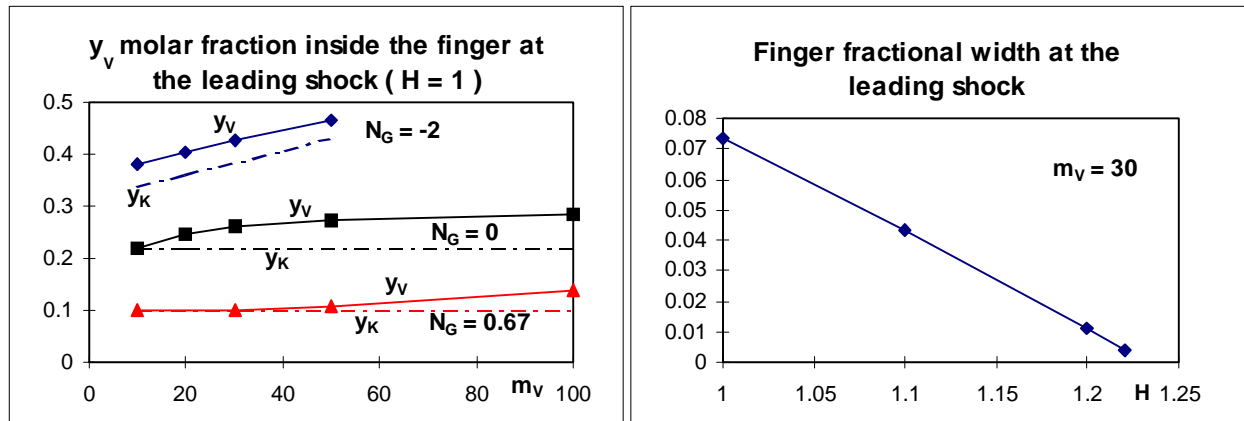
The corresponding fingering function can be expressed alternatively as a function of y_v or z_v :

$$26. \quad \Lambda_B(y_v) = \frac{1}{y_v m_k H (1 - y_v) + y_v - y_k} \quad \Lambda_B(z_v) = \frac{z_v}{(1 - y_k) F_k + y_k} .$$

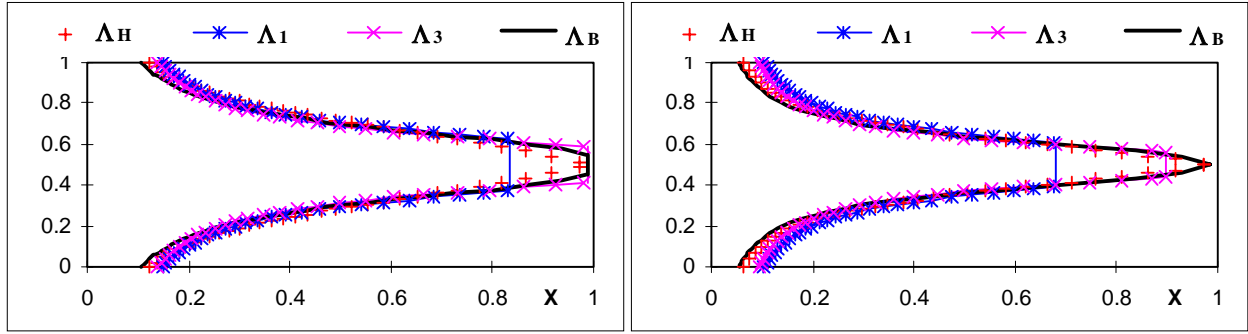
The V-component molar fractional flux in absence of gravity effect, F_v ($= F_g y_v$), is not formally equal to F_k :

$$27. \quad F_v = \frac{m_v H \Lambda_B(z_v)}{(m_g H - 1) \Lambda_B(z_v) + 1}, \quad \text{with} \quad m_g = [[(1 - y_k) F_k + y_k] (m_v^{1/4} - 1) + 1]^4 .$$

Contrary to the Koval function, a leading shock, depending on the viscosity ratio and the heterogeneity factor, can occur :



The various fingering functions, at $m_v = 30$, $N_G = 0$, are compared hereafter, on the left hand for $H = 1$ and $t = 0.35$ pvi, on the right hand for $H = 1.4$ and $t = 0.25$ pvi :



The new fingering function is relevant with the numerical experiments of Fayers et al. where bumps near the leading edge have been observed, for example at $H = 1$, $m_v = 30$, $N_g = 0$ and $N_g = -2$. Only Λ_B and Λ_H , which incorporate a dependence on the heterogeneity, are further compared. In the new one-phase two-component miscible model (named M_B), derived from using $\Lambda_B(z_v)$ in the FJT approach, z_v is a primary unknown, y_v and S_g are intermediate calculation variables.

It is also to be noted that F_v remains very similar to F_k on a large range of viscosity ratios and heterogeneity factors. Thus, the slope of F_v at $z_v = 0$ is formally equal to that of F_k , then to $(m_k H)$.

Extension of the Koval's model

One can use this particular feature ($F_v \approx F_k$) to derive, from the preceding equations, the following extension of the Koval's model, named 'K_B'. The V-component molar fractional flux is given by :

$$28. \quad Z_v = F_k (1 - \Gamma) \quad \text{with} \quad F_k = \frac{m_k H z_v}{(m_k H - 1) z_v + 1}, \quad \text{and} \quad \Gamma = N_g \frac{m_v - 1}{m_v} \frac{y_v (y_v - z_v)}{(H - 1) z_v + y_v}$$

where $y_v = (1 - y_k) F_k + y_k$ is an auxiliary variable, and y_k is given by eq.23. The total Darcy velocity (eq.18) is computed using the following effective average phase properties :

$$29. \quad \hat{\mu} = \mu_l \frac{(H - 1) z_v + y_v}{(m_g H - 1) z_v + y_v}, \quad \hat{\rho} = \rho_l - (\rho_l - \rho_v) F_k, \quad m_g \text{ from eq.22.}$$

The non-homogeneous single-phase physical properties are then found to depend on the heterogeneity.

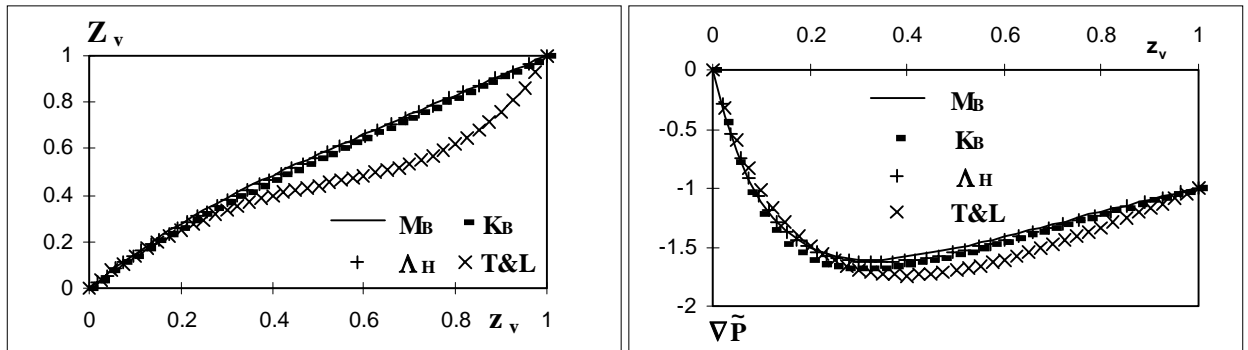
Model comparison

Both the comparison of the fractional flux function and the comparison of the pressure gradients are necessary to illustrate the behavior of the miscible models. The pressure gradients can be expressed in the following dimensionless form (a : two phases, two components, b : one phase two components) :

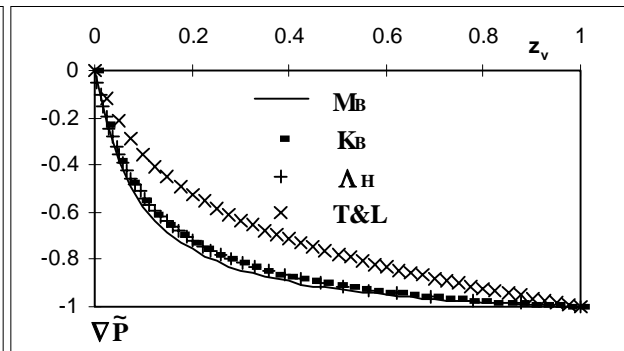
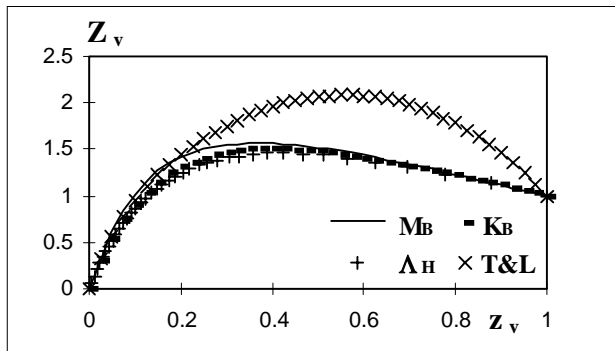
$$30. \quad \nabla \tilde{P} \stackrel{a}{=} \frac{1}{N_g - 1} \left[\frac{m_v}{m_v - 1} \left(1 - \frac{1 - \Gamma}{\mu_l \lambda_t} \right) - N_g \Delta \rho_n \right]; \quad \nabla \tilde{P} \stackrel{b}{=} \frac{1}{N_g - 1} \left[\frac{m_v}{m_v - 1} \left(1 - \frac{\hat{\mu}}{\mu_l} \right) - N_g \frac{\rho_l - \hat{\rho}}{\rho_l - \rho_v} \right]$$

In the further illustrations, are included the Todd & Longstaff model (with the ω value adjusted as in ref. 6), the FJT model, the K_B and M_B miscible models.

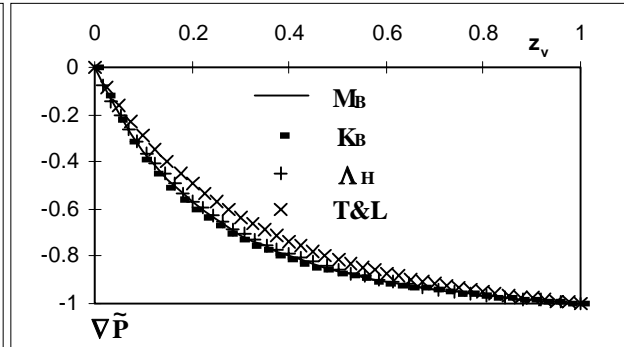
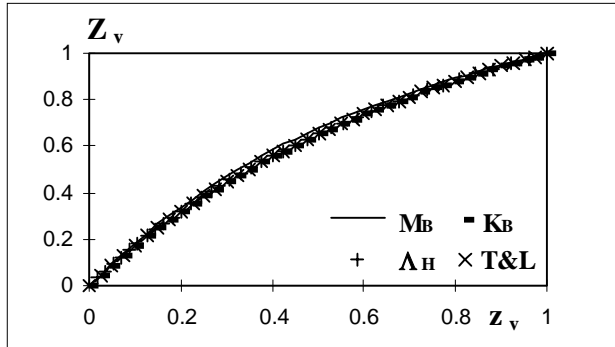
$$H = 1 ; m_v = 30 ; N_g = 0.67$$



$$H = 1.4 ; m_v = 30 ; N_g = -2$$



$H = 1.0 ; m_v = 10 ; N_g = 0$



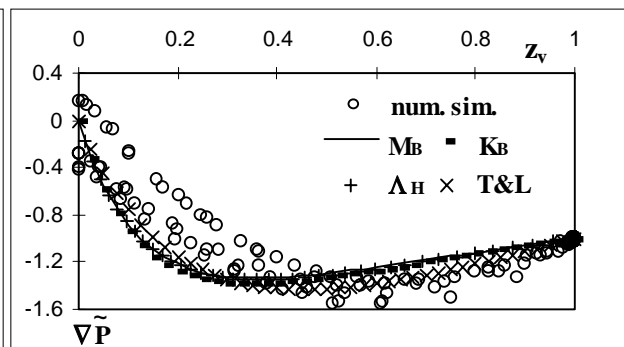
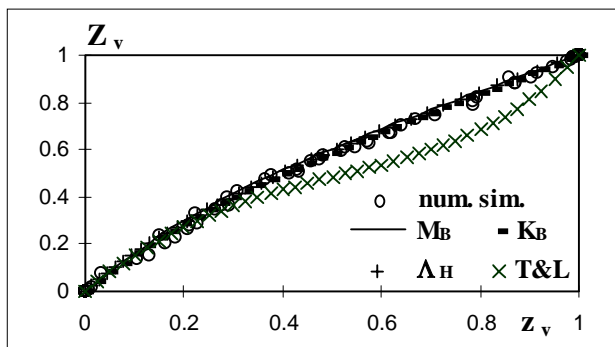
Although the number of examples shown here above is limited, the K_B and M_B models have been found very close to the FJT model on a wide range of dimensionless parameters (H , m_v , N_g).

Numerical simulations

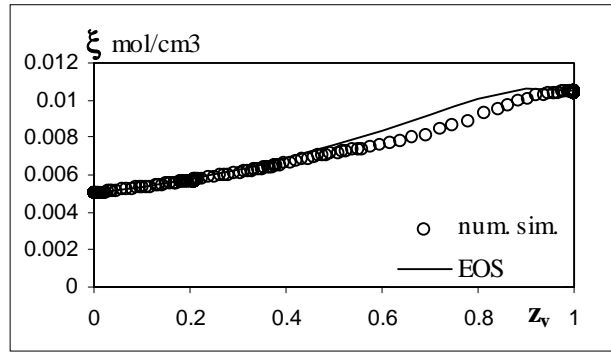
An industrial compositional simulator (ATHOS - BEICIP Franlab) has been used to simulate two-component first contact miscible displacements. Hereafter are illustrated the results of a gravity stabilized injection of methane into decane, at 121°C and 370 bar, in a cross-section (injection at the top, production at the bottom). In these thermodynamic conditions, the phase molar density (from Peng-Robinson equation of state) is nearly twice less for pure decane than for pure methane. The porous medium is slightly heterogeneous (permeability distribution is log normal with a reduced correlation length of 0.01 in both directions and $\sigma(\log k) = 0.7$); from a tracer test, H has been found equal to 1.15. The length to width ratio is equal to 3. The average permeability (1 Darcy) is high enough for limiting the local pressures to small variations. Hence, the compressibility effects are not very significant.

A good agreement is found between the molar fractional flux, from the numerical simulation, and from the FJT, M_B , and K_B models with the assumption of identical molar densities for the injected and in-place fluids. As the pressure gradients are very small, the magnitude of the relative error on ∇P is considerable and the results are scattered; the general trend is in better agreement with the T&L model than with the three other models. The observed tendency is not far different from that obtained by Fayers et al. for the simulation (with gas phase density from eq.14) of a gravity-stabilized displacement in a homogeneous medium at $N_g = 0.67$ and $m_v = 30$. It appears then that the assumption of equal molar sub-phases densities allows for deriving analytical models which give an acceptable approximation of the present case and of other simulated cases not reported here. This result may be not general.

$H = 1.15 ; m_v = 20.9 ; N_g = 0.61$



As shown below, the average phase molar density depends on the composition, and can be approximated using the bulk composition and the original PVT model (i.e. without changing EOS parameters) :



Implementation

In a compositional simulator, z_v is a primary unknown when solving the equations 17. The multi-component fluid representation is obtained using $z_k = x_k + z_v (y_k - x_k)$. The average phase molar density ξ is calculated with the standard PVT model using the z_k molar fractions. The phase density is obtained from eq.20. For the K_b model, the molar fractional flux function Z_v is taken from eq. 28., and the Darcy velocity is computed from eq. 18 and 29. For the M_b model, Z_v is derived from $Z_v = F_v (1 - \Gamma)$ (F_v from eq.27, 26, Γ from eq. 15,14) and the Darcy velocity is to be derived from eq. 18 and 19.

One way to implement the new models in a Black-Oil simulator is to transpose the equations 17 in a Black-Oil form (z_v, z_L represent the saturation variables and Z_v, Z_L the dynamic relative permeabilities) :

$$\partial_t (\phi \rho_g^* S_g^*) - \partial_x \left[\rho_g^* k \frac{kr_g^*}{\hat{\mu}} (\partial_x P - \hat{\rho} g \sin \varphi) \right] = 0 \quad \text{with} \quad S_g^* = z_v ; \quad kr_g^* = Z_v ;$$

$$\partial_t (\phi \rho_o^* S_o^*) - \partial_x \left[\rho_o^* k \frac{kr_o^*}{\hat{\mu}} (\partial_x P - \hat{\rho} g \sin \varphi) \right] = 0 \quad \text{with} \quad S_o^* = z_L ; \quad kr_o^* = Z_L .$$

ρ_g^* and ρ_o^* correspond respectively to $\rho_o^* = \xi MM_L$ and $\rho_g^* = \xi MM_v$; they have to be entered as functions of S_g^* . $\hat{\rho}$ and $\hat{\mu}$ keep the same definition as in the compositional implementation. Both phases have then differentiated fluid densities (ρ_g^* and ρ_o^*), but a same fluid viscosity and a same fluid density for the calculation of the Darcy velocity.

Conclusions

- 1- The transition from a two-phase multi-component model to a one-phase two-component model has been presented. It arises that the material balance equations have to be modified by the introduction of fractional molar flux functions, and a specific fluid density for the calculation of the Darcy velocity.
- 2- One-phase two-component models (M_b, K_b) for representing fully miscible displacements at unfavorable viscosity ratio have been derived from using a new fingering function in a Fayers type model. The K_b model is an extension of the Koval's model : effective phase properties are provided and the fractional flux functions are corrected to take into account the gravity effects.
- 3- The new models reproduce very closely the pressure drop and fractional flow obtained by Fayers et al. with their so-called ' Λ_H ' function for any mobility ratio, heterogeneity factor or gravity number. The effective phase properties of the single 'non homogeneous' phase are shown to depend on the heterogeneity factor, the effective phase behavior appearing then not to be of pure thermodynamic type.
- 4- Both models can be implemented either as a multi-component compositional model (one phase) or a Black-Oil model (two phases).

Nomenclature

k	:	absolute permeability	kr	:	relative permeability
ϕ	:	porosity	g	:	gravity acceleration
S	:	saturation	φ	:	angle between the horizontal and the flow directions
P	:	pressure	ξ	:	phase molar density
x,y,z	:	component molar fraction in the oil, gas, bulk phase			

$\rho, \hat{\rho}$: phase density, phase density in the potential evaluation of the one-phase model	<i>Subscripts</i>	
μ	: viscosity	o, g	: oil, gas
u	: Darcy velocity	L, V	: liquid, vapor component
λ	: phase mobility	i	: reference
Z	: component molar fractional flux	n	: normalized or reduced
F	: phase fractional flux	t	: total
MM	: molar mass	r	: accumulation term
		f	: convection term
		x, t	: position, time

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