

# Gradient Method and Bayesian Formalism Application to Petrophysical Parameter Characterization

ROGGERO, Frédéric, Institut Français du Pétrole  
GUERILLOT, Dominique, Institut Français du Pétrole

Paper presented at the 5th European Conference on the Mathematics of Oil Recovery, Leoben, Austria, 3–6 Sept. 1996

## **ABSTRACT**

Numerical models are routinely used today to analyze the performance of hydrocarbon reservoirs. However, the fit of the historical data has to take into account the initial geological knowledge to provide physical production forecasts, even if reservoir parameters are inherently uncertain over large parts of a field.

This article proposes a methodology to obtain an improved grid representation of the geological parameters and to quantify uncertainties after history matching. The goal is to obtain a physically matched model, taking into account the a priori knowledge of the reservoir. The proposed method is based on the gradient method coupled with an efficient optimization algorithm.

An objective function defined as an extension of the least squares technique is introduced as a history matching criterion. A priori information on the parameters is integrated through Gaussian probability density functions. During history matching, a statistical analysis based on the Bayesian formalism provides a posteriori information on the parameters with reduced uncertainties.

For quantifying the uncertainties on production forecasts, an algorithm is used to select directly, from all the possible realizations, extreme behaviour models. A production forecasting criterion is introduced to constrain the model in order to produce extreme forecasts belonging to a given probability domain.

## **INTRODUCTION**

History matching methods<sup>1,4</sup> are designed for constructing a numerical simulation model that optimally synthesizes all the available data, in order to forecast the dynamic behaviour of a reservoir over its total production period. The simulation tool is in fact a key factor in drawing up the field development plan, and for managing the available reserves over time.

The history matching phase is normally considered as time-consuming and costly in a reservoir engineering study. This is why a single simulation model is usually built to make production forecasts. This approach relies on the confidence granted to the simulation model, and implicitly presumes the uniqueness of the solution to the reverse problem.

Sensitivity studies with respect to simulation parameters can be conducted to try to pinpoint the dispersion of the production forecasts. The major sources of uncertainty must therefore be identified to test different models, and to obtain the corresponding predictions. Unfortunately, this type of simplified approach is not always sufficient to obtain a realistic quantification of the uncertainties.

Geostatistical methods have made tremendous progress in recent years and offer some answers to uncertainty forecasting. The processing of several equi-probable images of a reservoir helps to diminish the production forecasting uncertainties by incorporating a maximum of data on the geological model. These methods demand an effort of basic research to manage and classify the images according to their extreme behaviours, to incorporate the dynamic data in order to constrain the geostatistical simulations, or to constrain the images a posteriori using an inversion process<sup>5</sup>.

Also noteworthy is the growing determination of the oil companies to quantify the production forecasting uncertainties directly, in order to facilitate economic decisions and to minimize the production risks. Conventional reservoir engineering methods are not always an adequate answer to this concern<sup>6,7</sup>.

The methods proposed here are enhanced by this context, and are accordingly aimed at guiding the reservoir engineer in uncertainty forecasting. The general theoretical framework is based on the Bayesian formalism to propose a consistent and complementary approach to history matching techniques.

## **BAYESIAN INVERSION APPROACH**

The Bayesian inversion method<sup>8,10</sup> enables the simulation model to be seen in a stochastic form. An a posteriori uncertainty model is built by integrating all the available data, including the initial geological knowledge and the dynamic production data. This uncertainty model quantifies the predictive quality of the different simulation models by associating a probability level with each set of parameters. The a posteriori uncertainty model can therefore be considered as a synthesis of all the available data, both static and dynamic.

The contribution of the Bayesian inversion method for the history matching phase has already been demonstrated. The integration of the static and dynamic data is a satisfactory means of checking the coherence of these data. Since the parameter space is explored systematically, the volume of available information is very large. For instance, it is possible to identify multiple solutions or to quantify correlations between the parameters. Moreover, the initial knowledge of the parameters is memorised during the matching procedure, thus ensuring minimum deviation from physical reality (initial geological model).

To quantify the production uncertainties, an additional step is needed to predict the transfer of the models uncertainties to the production forecasts uncertainties. The Bayesian inversion method probabilizes the simulation model without a direct quantification of the impact on the forecasts. It is nevertheless possible to associate, with each numerical simulation, the probability of the corresponding model, obtained by Bayesian inversion.

### **Constraining the initial model**

The principle of Bayesian inversion consists in modifying an initial geological model by constraining it with the dynamic production data. The aim is to improve the quality of production forecasts by integrating the static and dynamic data.

The initial geological model, called the a priori model, is studied in a stochastic form. The reservoir is presumed to be described by a set of  $n_\theta$  parameters  $\theta$  with an associated uncertainty model, described by a probability density function  $f_\Theta(\theta)$ . Any probability function can be introduced. This a priori probability density function can be described by mean values  $\mu_\theta$  of the parameters and by the standard deviations  $\sigma_\theta$ . By mean of example, for a Gaussian uncertainty model, described by a covariance operator  $C_\theta$ , the a priori probability density function can be expressed as follows:

$$f_\Theta(\theta) = Cte \times \exp \left\{ -\frac{1}{2}(\mu_\theta - \theta)^T C_\theta^{-1}(\mu_\theta - \theta) \right\} \quad (1)$$

where:

$$Cte = \{2\pi^{n_\theta} \det C_\theta\}^{-1/2} \quad (2)$$

The dynamic production data are presumed known for a discrete set of  $t_i$  observation times. Let  $o$  be a set of  $n_o$

observations, or measurements, of the continuous physical quantity  $O$ . The measurements  $o$  are incorporated by constraining the a priori uncertainty model. The Bayes formula is used to obtain a posterior uncertainty model translating the conditional probability of obtaining the geological model, described by the parameters  $\theta$ , taking into account the observations  $o$ . The a posteriori probability density function (pdf) can be expressed as follows:

$$f_{O|\Theta}(\theta | O = o) = \frac{f_{O|\Theta}(O = o | \theta) \cdot f_\Theta(\theta)}{f(O = o)} \quad (3)$$

where:

$f_{O|\Theta}(O = o | \theta)$  is the probability of obtaining observations knowing the parameters  $\theta$ ,

$f_\Theta(\theta)$  is the a priori uncertainty model, and

$f(O = o)$  is a normalization term to obtain a pdf.

To apply this inversion principle to reservoir engineering, the numerical simulator is considered as a mathematical function, depending on the  $n_\theta$  parameters  $\theta$  and on time  $t$ , representing the stochastic physical quantity  $O$ , hence:

$$O = \omega(\theta, t) \quad (4)$$

The probability of matching the observations using the numerical model and knowing the parameters  $\theta$ , or the likelihood function, can then be expressed in the form:

$$f_{O|\Theta}(O = o | \theta) = f_{O|\Theta}(\omega(\theta, t) = o | \theta) \quad (5)$$

If the measurement errors and the errors associated with the numerical simulation are expressed by a Gaussian model, described by a covariance operator  $C_o$ , this likelihood function becomes:

$$f_{O|\Theta}(O = o | \theta) = Cte \cdot \exp \left\{ -\frac{1}{2}(o - O)^T C_o^{-1}(o - O) \right\} \quad (6)$$

where:

$$Cte = \{2\pi^{n_o} \det C_o\}^{-1/2} \quad (7)$$

This expression directly quantifies the differences between a given simulation and the measured values in probabilistic form. In practice, the complete space of the parameters must be scanned to obtain a pdf. It is therefore necessary to produce a very large number of numerical simulations, corresponding to each set of parameters  $\theta$ .

The Bayesian inversion procedure can be summarized as follows: for each set of parameters  $\theta$ , the associated probability density  $f_\Theta(\theta)$  is increased as the response of the model  $\omega(\theta, t)$  reaches the observations  $o$ . Hence the maximum a posteriori probability corresponds to a good compromise between the integration of the initial geological knowledge, described by  $f_\Theta(\theta)$ , and the matching of production data by the simulator, quantified by the likelihood function  $f_{O|\Theta}(O = o | \theta)$ .

## Forecasting production uncertainties

The objective is to predict the impact of the uncertainties on production forecasts. Using the Bayesian inversion method, each forecast can be associated with the a posteriori probability of the corresponding model. The analysis of the a posteriori model enables confidence intervals to be defined in the parameter space, or to observe the probability associated with given forecasts. This leads to two different methods, as described below:

- The first method consists of defining a confidence interval in the parameter space, bounded by a given probability criterion, and to select all the model realizations that are satisfactory to this confidence criterion. The envelope of the simulated forecasts from all these realizations quantifies an uncertainty margin for a given confidence criterion.
- The second method is to consider a given time and to observe the probability associated with each realization of the model as a function of the production forecasts. To quantify the production uncertainties, the models giving extreme forecasts for a given probability value are considered.

The major drawback of this procedure is its prohibitive cost, due to the quantity of simulations required. It is therefore necessary to map the complete parameter space in order to obtain the a posteriori pdf. The search for a normed function, by definition a probability density function, demands integration over a sufficiently broad range of parameters to cover the entire uncertainty domain. For each set of parameters, a direct simulation is required to assess the likelihood function.

For this reason the procedure is often, in practice, limited to searching for the maximum a posteriori probability. This means setting aside the actual value of the a posteriori pdf, in order to locate the optimum given by the parameters values. This search can be conducted automatically by using an appropriate optimization algorithm.

Yet quantification of production uncertainties demands the input of additional numerical simulations, based on different realizations of the model: the inversion procedure limited to the search for the maximum likelihood can in effect give an average estimate of the forecasts, without any information on the variance of these forecasts. A consequential compromise must therefore be found between the systematic exploration of the uncertainty domain, demanding a large number of realizations, and a search for a unique model matching the data, but only producing a unique forecast.

The aim of the production scenario test method<sup>11</sup>, proposed in the next Section, is to select different simulation models corresponding to given production assumptions. This makes it possible to search for the extreme models directly, in order to quantify production uncertainties.

## PRODUCTION SCENARIO TEST METHOD

A complementary approach to history matching methods is therefore needed to characterize the quality of production forecasts and to quantify the uncertainties. The aim here is to predict the extreme behaviours of different models and to directly quantify the production forecasting uncertainties, by incorporating all the available data.

Stochastic methods can be considered for a detailed quantitative analysis of the uncertainty domain. This can be achieved by a Monte Carlo approach, wiping out a large number of numerical simulations allowing the probabilization of the production forecasts. The stochastic approach is unfortunately often impractical in reservoir engineering, owing to its high cost.

Attempts can be made to reduce the number of simulations by directly searching the extreme behaviour models, for a given production forecasting criterion. This avoids the necessity to scan all the domain of possible parameters, by selecting combinations of critical parameters.

The proposed approach is based on the production scenario test. This method consists of testing different models traducing production forecast hypotheses, by adding new data to the actual production data. The data added to create scenarios represent constraints, and physically correspond to the imposition of a search direction for a new model.

These data can be incorporated in the initial model by a conventional inversion procedure. For each scenario, a new geological model is obtained, reflecting the constraints imposed, and a direct numerical simulation gives a new forecast.

The scenario test method can thus be used to select simulation models corresponding to production forecast assumptions. The probability of each of these models can be quantified directly, using the Bayesian inversion method, by the a posteriori pdf.

To quantify the production forecasting uncertainties, the method can be applied to searching for extreme production scenarios. The management of the uncertainties at a given future time  $t_f$  amounts to selecting, from the possible models, the parameters corresponding to a high or low forecasting criterion. By referring to the Bayesian formalism, an isoprobability criterion can be defined to find these extreme models.

The application of the scenario test method to find extreme forecasts can be reduced to the solution of two optimization problems corresponding to an 'optimistic' scenario and to a 'pessimistic' scenario.

- Search for the optimistic scenario: identification of the parameters that maximize the production forecasting criterion with the best possible respect of the history matching:

Min(Matching Criterion) + Max(Forecasting Criterion)

- Search for the pessimistic scenario: minimum value of production forecasting criterion compatible with history matching:

Min(Matching Criterion) + Min(Forecasting Criterion)

A tolerance margin can be set to meet the history matching criterion, reflecting agreement with the initial geological data and with the historical data. If the definition of the matching criterion helps to interpret this tolerance margin as a probability criterion, the uncertainty can be quantified from the difference between the extreme forecast values, for a given probability criterion.

The definitions of the history matching and production forecasting criteria, needed for the application of the method, are covered in next Section. These criteria are described as objective functions, in order to treat the problem of quantifying uncertainties as one of optimization. Specific optimization algorithms will be proposed to solve this problem.

## History Matching Criterion

The aim is to be able to compare different simulation models, corresponding to different parameters choices, by quantifying the uncertainties or the predictive quality of each of these models. The history matching criterion quantifies the differences between the observed values and the results of a simulation. Solving the reverse problem is therefore equivalent to identifying the parameters of the simulation model to obtain the best possible criterion.

The objective function proposed offers a flexible formulation to account for different terms depending on the type of problem treated: correlated or uncorrelated measurements, possibly of different types, a priori information given by means or correlations between parameters. A generalization of the weighted least-squares criterion is used, by introducing matrix operators in the space of observations, denoted  $C_o^{-1}$ , and in the space of parameters, denoted  $C_\theta^{-1}$ . The expression of this objective function is the following:

$$F(\theta) = F_o(\theta) + F_\theta(\theta) \quad (8)$$

with:

$$\begin{aligned} F_o(\theta) &= \frac{1}{2}(o - O)^T C_o^{-1}(o - O) \\ F_\theta(\theta) &= \frac{1}{2}(\mu_\theta - \theta)^T C_\theta^{-1}(\mu_\theta - \theta) \end{aligned} \quad (9)$$

By referring the Bayesian formalism, the operators  $C_o$  and  $C_\theta$  can hence be interpreted as covariance operators describing the uncertainties in the measurement space or in the parameter space. The link between the Bayesian formalism and the choice of objective function<sup>10</sup> is clearly established by the following equations:

$$\begin{aligned} f_{\Theta|O}(\theta | O = o) &= Cte. \exp \{-F(\theta)\} \\ f_\Theta(\theta) &= C_1. \exp \{-F_\theta(\theta)\} \\ f_{O|\Theta}(O = o | \theta) &= C_2. \exp \{-F_o(\theta)\} \end{aligned} \quad (10)$$

It is strictly equivalent to find the parameters that minimize the objective function  $F(\theta)$  or to find the parameters corresponding to the maximum a posteriori pdf. This enables use of optimization methods with a probabilistic interpretation of the results. It must nevertheless be observed that optimization methods cannot be used to calculate the constant *Cte* coefficient, because this demands a complete integration of the parameter space. The different assessments of the objective function  $F(\theta)$  are in fact only useful for comparing relative values of the a posteriori pdf.

## Production Forecasting Criterion

The direct link between the matching criterion and the posterior uncertainty model introduces a concept of an admissible probability domain for selecting the parameters. In fact, a given matching criterion  $F_{hmc}$  corresponds to the definition of an isoprobability contour through the following equation:

$$\begin{aligned} F(\theta) &= F_{hmc} \Rightarrow \\ f_{\Theta|O}(\theta | O = o) &= Cte. \exp(-F_{hmc}) \end{aligned} \quad (11)$$

Since the posterior pdf is known, in most cases, to within a constant value, it is easier to introduce a relative posterior probability criterion, compared to a reference value. Let the posterior probability ratio *ppr* with respect to the maximum probability  $f_{\Theta|O}(\theta_\infty | O = o)$  be defined as follows:

$$ppr = \frac{f_{\Theta|O}(\theta | O = o)}{f_{\Theta|O}(\theta_\infty | O = o)} \quad (12)$$

where  $\theta_\infty$  is the location of the optimum. The history matching criterion therefore becomes:

$$F_{hmc} = F(\theta_\infty) - \log(ppr) \quad (13)$$

A given *ppr* ratio delimits a minimum probability domain for selecting the models with the following equations:

$$\begin{aligned} f_{\Theta|O}(\theta | O = o) &\geq ppr \times f_{\Theta|O}(\theta_\infty | O = o) \\ \Leftrightarrow F(\theta) &\leq F_{hmc} \end{aligned} \quad (14)$$

The production uncertainties can be quantified by the envelope of production forecasts simulated from all the realizations of the model belonging to a given probability domain. The production scenario matching method enables direct selection, from all these possible realizations, of the models corresponding to production forecast assumptions.

To find the extreme production forecasting scenarios, the search can be limited to models that minimize or maximize a given production forecast criterion. This forecast criterion defines the variables analyzed from the simulation results, which may, for example, be the final recovery rate, cumulative oil production, or any other criterion based on economic considerations.

The choice of the forecasting criterion determines the selection of the models within the probability domain. This selection therefore depends on the type of defined variable and on the observation time(s) of this variable. In particular, different observation times can modify the selection of the extreme models.

Let  $O_f$  be the stochastic variable concerned by the uncertainty analysis, and  $o_f$  a set of observations of this variable at given production forecasting times  $t_f$ . Note that the forecasting criterion may be different from the quantities observed for history matching.

The numerical simulation model, matched to the production data, gives a preliminary estimate  $o_{f\infty}$  of the mean of the variable  $O_f$  :

$$o_{f\infty} = \omega_f(\theta_\infty, t_f) \quad (15)$$

where  $\omega_f$  is the function used to calculate the production forecasting criterion from the numerical simulation model  $\omega$ . To quantify extreme production forecasts, models must be found belonging to the confidence domain that maximize or minimize the criterion predicted by simulation  $\omega_f$  at the given forecasting times  $t_f$ .

To do this, new data  $o_f$  can be introduced at future times  $t_f$  to distort the production forecasts in the desired direction. These artificial data are new constraints superimposed on the history matching problem. The values of these constraints, which are initially unknown, must be adjusted until a minimum or a maximum is obtained at  $t = t_f$ . The criterion to be minimized can be defined by a new term  $F_f$  included in the objective function definition:

$$F_f = \frac{1}{2} w_f (o_f - \omega_f(\theta, t_f))^2 \quad (16)$$

where:

- $F_f$  : term of constraint by forecasting criterion,
- $w_f$  : weight coefficients (inverses of variances of  $o_f$ ),
- $o_f$  : data added at future time(s)  $t_f$ ,
- $\omega_f(\theta, t_f)$  : production forecasts at time(s)  $t_f$ .

The proposed formulation has the advantage of reducing the quantification of production uncertainties to a matching problem of data including historical data, the a priori knowledge, and the data added as constraints. The objective function including all these constraints is written as follows:

$$\begin{aligned} F(\theta) &= F_o(\theta) + F_\theta(\theta) + F_f(\theta) \\ F(\theta) &= \frac{1}{2} (o - O)^T C_o^{-1} (o - O) \\ &+ \frac{1}{2} (\mu_\theta - \theta)^T C_\theta^{-1} (\mu_\theta - \theta) \\ &+ \frac{1}{2} w_f (o_f - \omega_f(\theta, t_f))^2 \end{aligned} \quad (17)$$

Optimization methods are proposed in next Section to deal with the problem of minimizing this objective function  $F(\theta)$  with or without extreme forecasts constraints. These algorithms are particularly appropriate for minimizing a least squares type of objective function.

## OPTIMIZATION TOOL

An optimization tool has been developed to identify the optimal parameters in an automatic iterative process. The principle is to calculate an improved estimation of the parameters after each simulation, in order to minimize the objective function. At each iteration, a new simulation is run with the new parameters values. The process stops when a convergence criterion is reached.

The main components of this optimization tool are:

- The ATHOS forward numerical simulator
- The gradient method
- The optimization loop

The gradient method<sup>12-15</sup> implemented in a multipurpose reservoir simulator can be used for multiphase or monophasic applications. It enables computation of the derivatives of the main production results with respect to the descriptive parameters of the simulation model. The available parameters are porosities or permeabilities assigned to given reservoirs zones, transmissivities and well productivity indexes. Multiplying factors can be used for inverting heterogeneous distributions of petrophysical properties.

Analysis of the gradient values helps to carry out conventional sensitivity studies effectively, and provides an interesting aid for the optimal selection of the inversion parameters<sup>16</sup>. The use of gradients considerably enhances the performance of the optimization algorithms and helps to consider matching aid methods or automatic matching methods.

The reverse modelling tool applies to both history matching and production forecasting applications in an integrated approach. Different optimization algorithms<sup>10,17-19</sup>, based on the use of gradients, are proposed to solve these problems efficiently:

- The steepest descent method is well known for its numerical robustness, and is adapted for starting far from the solution. The major drawback of this method is a large deterioration of the convergence rate as the solution is approached.
- The Gauss-Newton algorithm is highly efficient in most cases, and approaches a quadratic convergence rate. This method may unfortunately become unstable in dealing with a highly non-linear problem or if the initial parameters are too different from the optimal solution.
- The Levenberg-Marquardt algorithm is an improvement of the Gauss-Newton method obtained by introducing a regularization term in the Hessian matrix. A better numerical stability is obtained and quasi-singular matrix problems are avoided.
- The dog-leg method, proposed by Fletcher and Powell, combines the steepest descent and the Gauss-Newton solutions efficiently, for obtaining both numerical robustness far from the solution and rapid convergence near the solution.

## SPE1 TEST CASE

A synthetic case is defined to validate the proposed approaches for history matching and quantifying production uncertainties. A reference simulation model is used to build synthetic data including historical data and geological knowledge of the reservoir.

The history matching phase enables the parameters of the simulation model to be constrained by the dynamic production data. An optimization algorithm is used to obtain an a posteriori model which matches the production data while accounting for the initial geological knowledge.

The final phase of the study concerns the production forecasting and the quantification of the uncertainties. The scenario test method is evaluated for finding extreme forecasts for a given future time. The uncertainty forecasts are validated by comparison with the reference simulation.

## Reference case description

The synthetic case is an adaptation of the SPE1 case. The reference model represents a five-spot configuration on a 10,000 ft sided reservoir portion. Two wells (gas injector and producer) are located at the opposite corners of the reservoir. The injection well is drilled through the entire upper geological layer, and the producing well through the entire lower layer.

The reservoir consists of three horizontal layers forming distinct geological units (Fig. 1). The permeabilities are distributed heterogeneously in each of these layers. The interfaces between the layers form two permeability barriers with heterogeneous properties. The other petrophysical properties are constant throughout the reservoir.

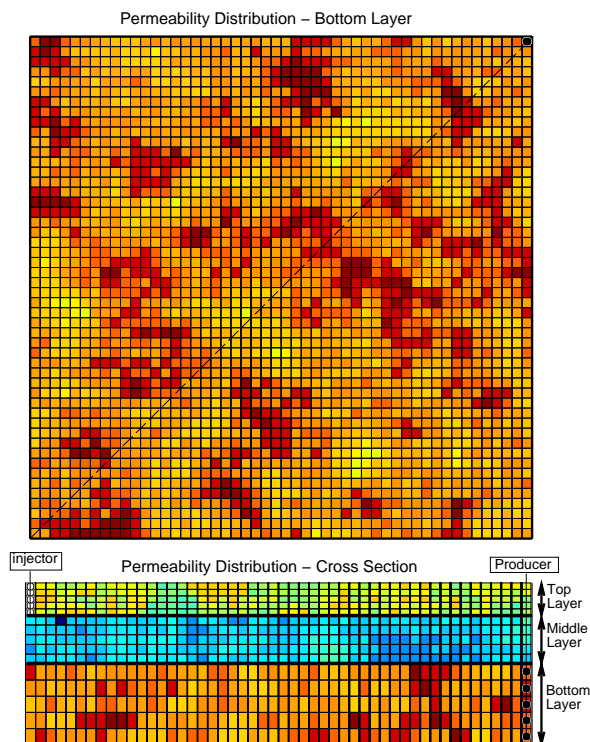


Fig. 1: Reference case description

The initial pressure of the reservoir is 5800 psi. The bubble point of the oil is 4014.7 psi. The initial water saturation is 12%. The water phase is presumed to be immobile, with a zero relative permeability curve. The water/oil and gas/oil capillary pressures are also zero.

The producing well is controlled by a bottom hole pressure limited to 4100 psi and a maximum oil flow of 30,000 bbl/day in surface conditions. The gas injection rate is 65,000 Mscft/day with a maximum bottom hole pressure of 8500 psi.

The reference model consists of a  $50 \times 50 \times 15$  grid, with a constant step of 20 ft in the horizontal X and Y directions. Each geological unit is modelled by five cell layers of the same thickness, with a lognormal distribution of the permeabilities (parameters  $K_{top}$ ,  $K_{mid}$  and  $K_{bot}$ ). The geostatistical simulation parameters used are listed in Table 1. The inter-layer permeability barriers are modelled by a lognormal distribution of multiplication factors of vertical transmissivities (parameters  $MTZ_{top}$  and  $MTZ_{bot}$ ), with the means given in Table 1.

Table 1: Distribution of petrophysical properties

parameter	log mean	log stand. deviation	parameter mean
$K_{top}$	2.3	0.115	200 mD
$K_{mid}$	1.7	0.085	50 mD
$K_{bot}$	2.7	0.135	500 mD
$MTZ_{top}$	-0.194	0.05	0.64
$MTZ_{bot}$	-0.585	0.05	0.26

A spherical variogram is used for the three simulations in each of the layers. The correlation lengths are 1000 ft for the horizontal directions and 250 ft for the vertical direction. Note that these correlation lengths are relatively short in comparison with the inter-well spacing, and that the permeability contrasts are weak.

The synthetic production data are produced by a numerical simulation using the ATHOS model over a ten-year period. The production results over the total duration of the simulation period are shown in Fig. 2 to 4.

The injection well operates in flow rate conditions that are imposed for the entire simulation, while the producing well reaches the bottom hole pressure limit from the onset of the simulation. The gas breakthrough in the producing well occurs after 1125 days.

The synthetic historical data are defined from the reference numerical simulation. Only the period preceding the gas breakthrough time has been selected. The history matching therefore concerns this period, and the subsequent period is used as a reference to predict the production uncertainties. The production results selected as synthetic measurements are the oil and gas flow rates at the producing well ( $Q_{oil}$  and  $Q_{gas}$ ) and the bottomhole pressure in the injection well ( $P_{wf}$ ).

The uncertainties associated with these observed values are represented by a Gaussian model centered on the observed values, with standard deviations of 2.5% of each of these values.

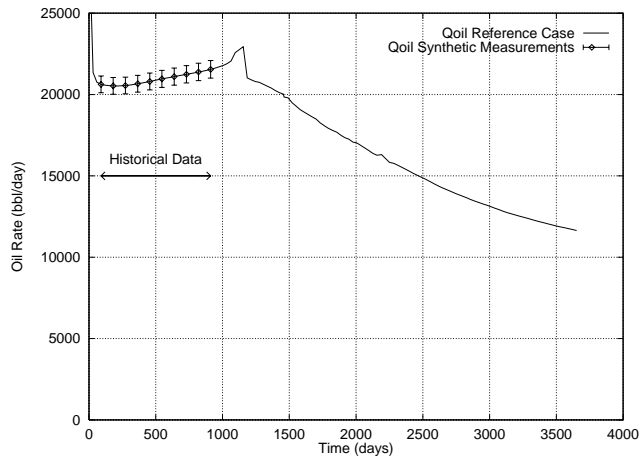


Fig. 2: Reference simulation: Oil rate

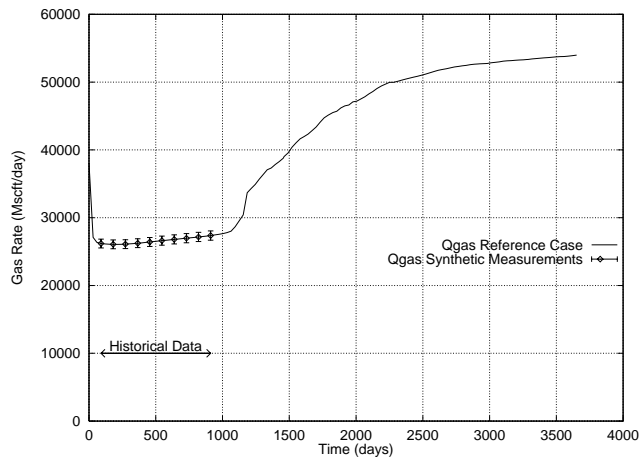


Fig. 3: Reference simulation: Gas rate

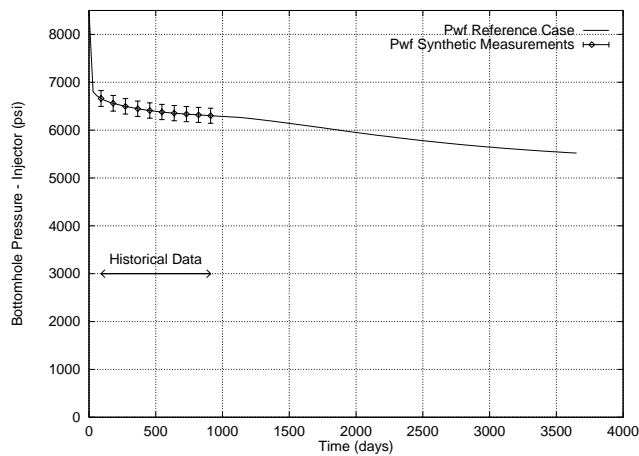


Fig. 4: Reference simulation: bottomhole pressure at injector

### Simulation model definition

The geometric data, the geological structure, the fluid properties and the relative permeabilities are presumed known, and correspond to the data of the reference case. Uncertain data of the model are the absolute permeabilities, the vertical exchange coefficients between the geological layers, and the well productivity indexes.

Multiplying factors for the well productivity indexes ( $MPI_{prod}$  and  $MPI_{inj}$  parameters) are introduced to translate the uncertainty on the well/reservoir connection. These parameters also enable to correct for the homogenization of the permeabilities found at the perforations.

The a priori knowledge of the simulation model is synthesized by a Gaussian uncertainty model, defined by the data of the means and standard deviations of each parameter. The parameters are shown in Figure 5 and the corresponding values are listed in Table 2 below:

Table 2: A priori simulation model parameter

parameter	a priori mean	a priori standard deviation
$K_{top}$	350 mD	100 mD
$K_{mid}$	50 mD	20 mD
$K_{bot}$	350 mD	100 mD
$MTZ_{top}$	1	0.5
$MTZ_{bot}$	1	0.5
$MPI_{prod}$	1	0.5
$MPI_{inj}$	1	0.5

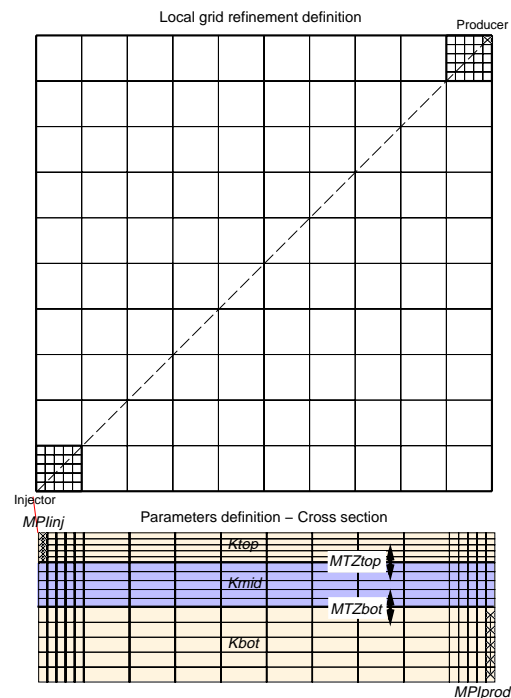


Fig. 5: Simulation model and parameters definition

The simulation model is built with a  $10 \times 10 \times 15$  grid. The vertical definition of the reference model has been preserved (Figure 5). Since the correlation lengths are relatively short, uniform petrophysical parameters are used for each geological unit.

Initially the grid effect is analyzed by comparing the simulation results obtained on the coarse grid, using the reference values of the parameters, and the simulation results on a fine  $50 \times 50 \times 15$  grid with uniform permeability distributions. The oil production curves (Figure 6) show a non-negligible effect of the grid.

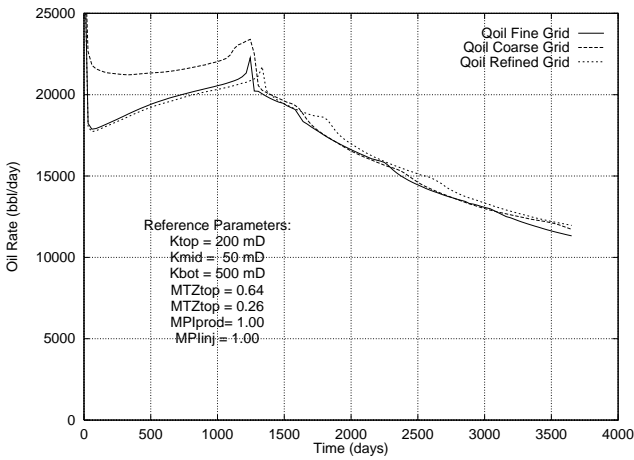


Fig. 6: Grid effect on the oil rate

The grid is refined at the wells to correct this numerical effect: the cells corresponding to the well locations are replaced by a  $5 \times 5 \times 15$  subgrid along the entire height of the model (Figure 5). The results obtained with grid refinement are close to the results obtained on the fine grid throughout the simulation (Fig. 6), with a slightly delayed gas breakthrough time (about 100 days).

### Sensitivity study by the gradient method

An initial numerical simulation is conducted from the a priori means of the parameters, defined in Table 2. The gradient method is applied simultaneously, to compare the sensitivities of the production forecasts to the parameters of the model.

The gradients of the oil flow rate and the bottom hole pressure in the injection well are shown in Figures 7 and 8. The gradients are multiplied by the actual value of each parameter, to obtain homogeneous quantities for each variable. By means of example, the gradient of the bottom hole pressure with respect to the  $K_{top}$  permeability is uniform at a pressure, and represents the effect of a 100% variation in permeability.

The most sensitive parameters are the  $K_{top}$  and  $K_{bot}$  permeabilities. In comparison, sensitivity to the permeability of the middle layer  $K_{mid}$  is much lower. The inter-layer liaison  $MTZ_{top}$  and  $MTZ_{bot}$  parameters have very little effect on the oil flow rate and on the bottom hole pressure. The effect of the productivity index of the producing well ( $MPI_{prod}$  parameter) is significant on the

oil flow rate, but is also very clear on the bottom hole pressure of the injection well. The productivity index of the injection well ( $MPI_{inj}$  parameter) only affects the bottom hole pressure.

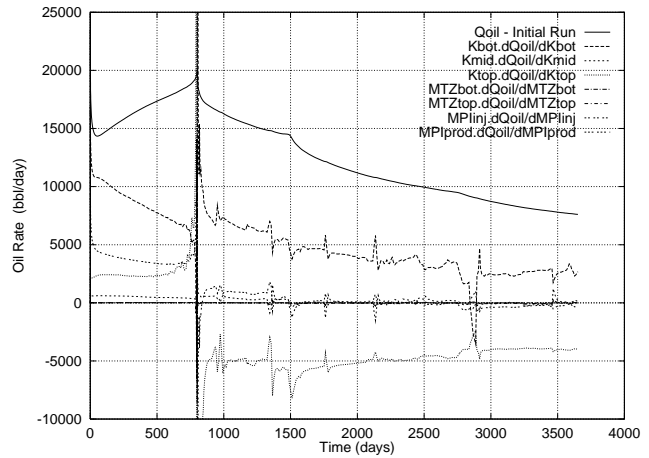


Fig. 7: Oil rate gradients with respect to the parameters

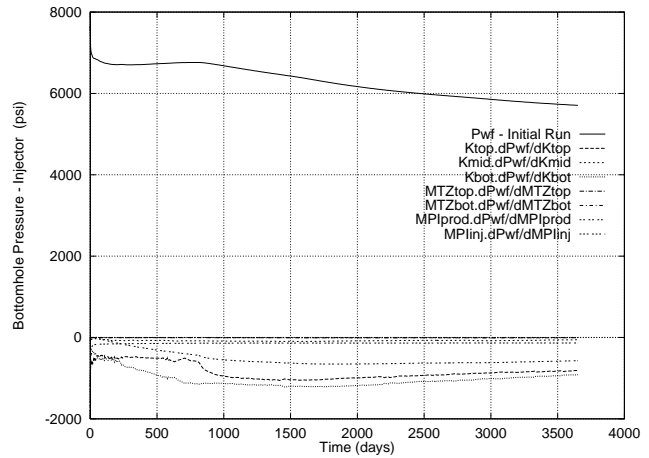


Fig. 8: Bottomhole pressure gradients with respect to the parameters

$K_{top}$ ,  $K_{bot}$  and  $MPI_{prod}$  are therefore the prominent parameters and, to a lesser degree,  $K_{mid}$  and  $MPI_{inj}$ . These results reveal the possibility of ignoring the influence of the inter-layer liaison parameters for history matching. However, all seven parameters are preserved for the rest of the study, to ensure wider forecast variability for quantifying uncertainties. It is in fact important to check whether wide ranges of variations in the parameters, initially with very little influence, cause fundamental changes in the dynamic behaviour of the reservoir.

### History Matching

For history matching, the parameters of the a priori simulation model are constrained by the dynamic production data. An optimal matching is sought corresponding to maximum a posteriori probability, by minimization of

an objective function defined as follows:

$$F(\theta) = \frac{1}{2}(o - O)^T C_o^{-1}(o - O) + \frac{1}{2}(\mu_\theta - \theta)^T C_\theta^{-1}(\mu_\theta - \theta) \quad (18)$$

where:

$O = \{Q_{oil}, Q_{gas}, P_{wf}\}$ , vector formed by the variables simulated at the observation times,

$o = \{Q_{oil}^o, Q_{gas}^o, P_{wf}^o\}$ , vector of historical data measurements,

$C_o$  = diagonal matrix whose elements are the variances of the values observed,

$\theta = \{K_{top}, \dots, MPI_{prod}\}$ , vector of the 7 parameter values,

$\mu_\theta$  = vector of a priori means of the parameters,

$C_\theta$  = diagonal matrix whose elements are the a priori variances of the parameters.

All the available data are taken into account for the matching criterion  $F(\theta)$ . The first term  $(o - O)^T C_o^{-1}(o - O)$  corresponds to the search for an optimal match of the historical data. The three measurements, which are of different types, are matched simultaneously: the oil flow rate  $Q_{oil}$ , the gas flow rate  $Q_{gas}$ , and the bottom hole pressure in the injection well  $P_{wf}$ . These measurements are normed by the covariance operator  $C_o$ .

The second term of the objective function  $(\mu_\theta - \theta)^T C_\theta^{-1}(\mu_\theta - \theta)$  is used to model the a priori uncertainty model: the vector  $(\mu_\theta - \theta)$  reflects the difference between a realization of the model, defined by the parameters  $\theta$ , and the a priori means  $\mu_\theta$ . The a priori variances form the diagonal of the covariance matrix  $C_\theta$ .

The Fletcher-Powell optimization algorithm is used to minimize the matching criterion  $F(\theta)$  in an automatic matching process. The choice of this method is a good compromise between the search for a robust algorithm and a minimum number of simulations.

The initial parameters correspond to the means of the a priori model. The convergence of the matching procedure is obtained in five iterations. The changes in the simulation results during the iterations are shown in Figures 9 to 11, for the oil flow rate (Figure 9), gas flow rate (Figure 10) and the bottomhole pressure of the injection well (Figure 11).

A good matching of all the measurements is obtained. This result can be checked on a plot of the variation in the matching criterion during the iterations (Figure 12). Note a sharp decrease in the objective function from the two first iterations. The breakdown of the criterion into different terms (Figure 12) shows the prominent weight of the a priori model term. The bottom hole pressure in the injection well is respected perfectly, with a very low residue. In comparison, the residues on the match of the oil and gas flow rates are slightly higher.

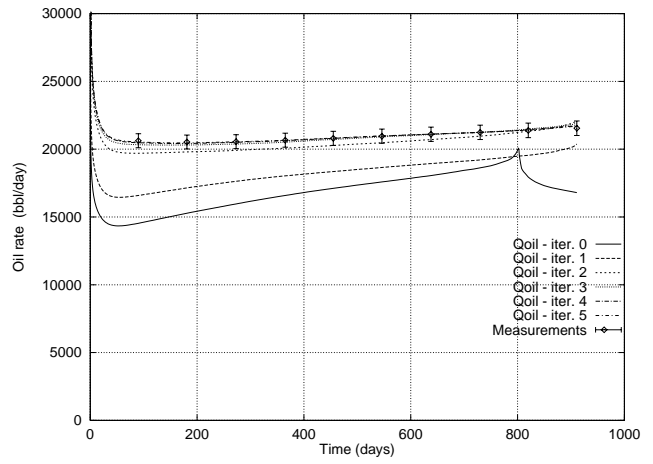


Fig. 9: Oil rate evolution during the optimization

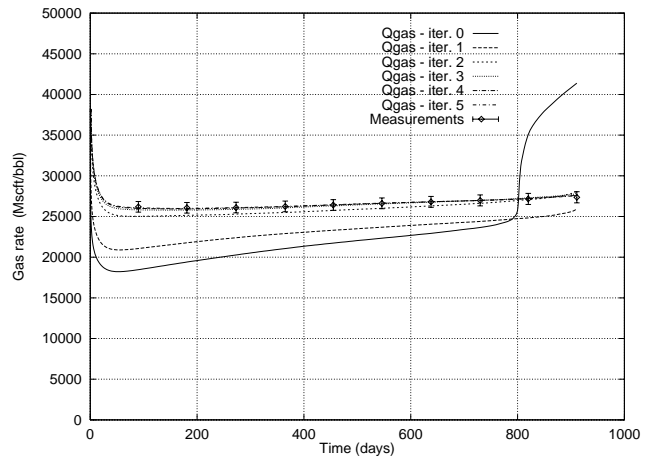


Fig. 10: Gas rate evolution during the optimization

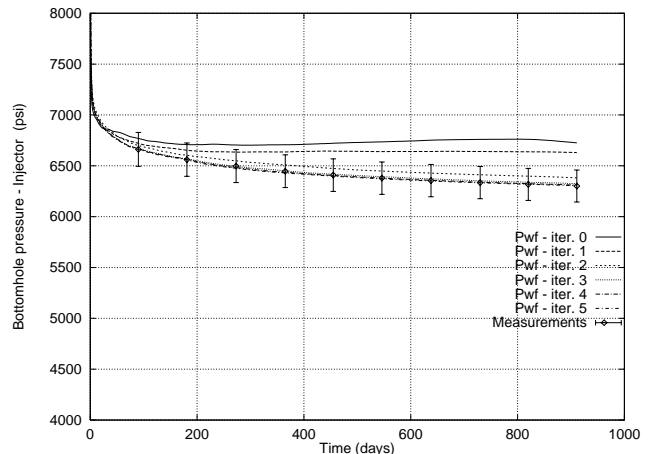


Fig. 11: BHP evolution during the optimization

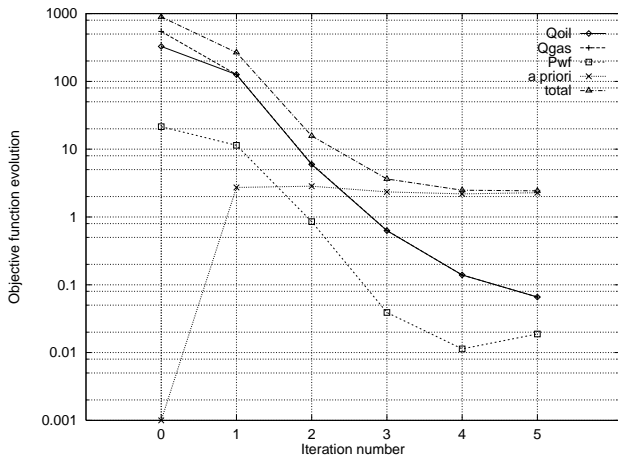


Fig. 12: Objective function evolution

Note that these results, which reveal a severe constraint of the a priori model, could lead to questioning the initial model. It is in fact possible to decrease this constraint by increasing the a priori standard deviations on the parameters, or by recentering the a priori means on the optimal values.

Visualization of the parameters during the matching process (Figure 13) shows that the main changes concern the permeabilities of the top and bottom layers ( $K_{bot}$ ,  $K_{top}$ ) and the well productivity indexes ( $MPI_{prod}$ ,  $MPI_{inj}$ ). The other parameters have practically not varied, and are therefore essentially characterized by the initial knowledge, confirming the conclusions of the sensitivity study.

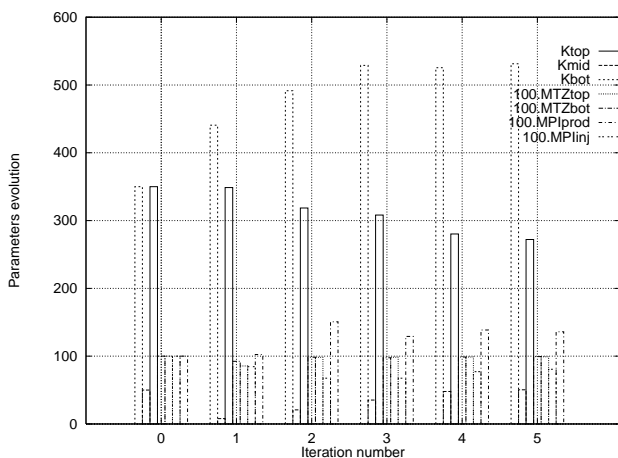


Fig. 13: Parameters evolution during the optimization

The simulation results obtained over a ten-year period using the optimal model are shown in Figures 14 to 16. These initial production forecasts are compared with the reference numerical simulation. A good match is obtained for the period corresponding to the historical data (0 to 911 days). Over the forecasting period, an error of about 100 days may be noted in the forecast of the gas breakthrough time. This error results in an underestimation of the oil production (of about 12% at the end of simulation).

## Scenario test method: extreme production forecasts

The history matching phase has served to identify the most probable model, as a function of the historical data available and the initial knowledge of the reservoir. A direct simulation from this model enables an estimation of the initial production forecasts over a ten-year period (Fig. 14 to 16). To quantify the uncertainties on these forecasts, it is necessary to predict the impact of the parameter estimation error on a given production forecast criterion.

The production forecast criterion examined is the oil flow rate at the end of simulation ( $t_f = 3652$  days). The scenario test method is used to identify the models giving extreme forecasts within a confidence domain, bounded by a minimum a posteriori probability criterion. Note that the probability criterion corresponds to a tolerance on the match of the historical data and on the respect of the a priori model.

In a first phase, a minimum probability criterion is set corresponding to 10% of the maximum probability, or a *ppr* ratio of 0.1. From the model corresponding to the optimal matching the optimization algorithm is conducted to minimize or to maximize the production forecast criterion, by constraining the model within the confidence domain.

For each of the min/max forecasts, convergence is obtained in seven iterations. Note that the cost in terms of the number of simulations could be reduced by decreasing the accuracy required for the compliance with the probability norm. This makes it possible to limit the number of iterations to two or three, and to recalculate the probability criterion actually obtained.

The parameters of the extreme models obtained and the corresponding forecasts are shown in Table 3 below:

Table 3: Models and extreme forecasts

parameter	min $Q_{oil}$ forecast	optima matching	max $Q_{oil}$ forecast
$K_{top}$	330.6 mD	272.1 mD	181.1 mD
$K_{mid}$	22.9 mD	50.4 mD	51.5 mD
$K_{bot}$	537.7 mD	531.3 mD	567.8 mD
$MTZ_{top}$	1.003	0.991	0.911
$MTZ_{bot}$	0.966	0.993	1.007
$MPI_{prod}$	1.260	1.360	1.332
$MPI_{inj}$	0.660	0.806	1.200
$Q_{oil}$ forecast	9246 bbl/d	10246 bbl/d	13136 bbl/d

The production forecasts obtained from these models are shown in Figures 14 to 16. The comparison of the uncertainty envelope bounded by the min/max forecast against the reference numerical simulation validates the results.

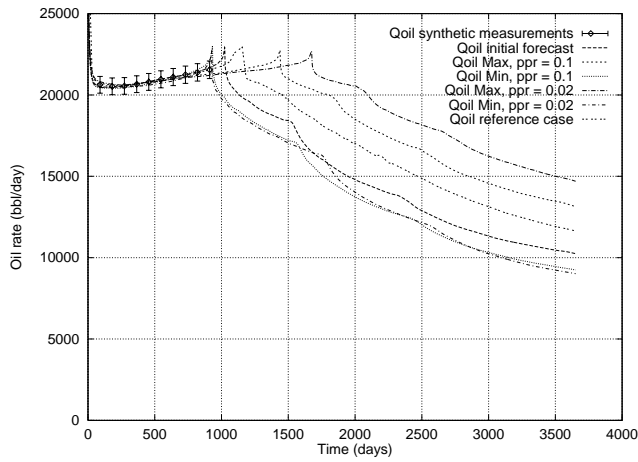


Fig. 14: Oil rate extreme forecasts

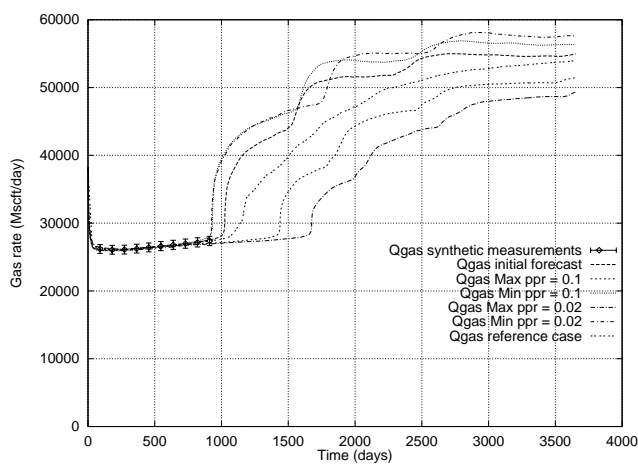


Fig. 15: Gas rate extreme forecasts

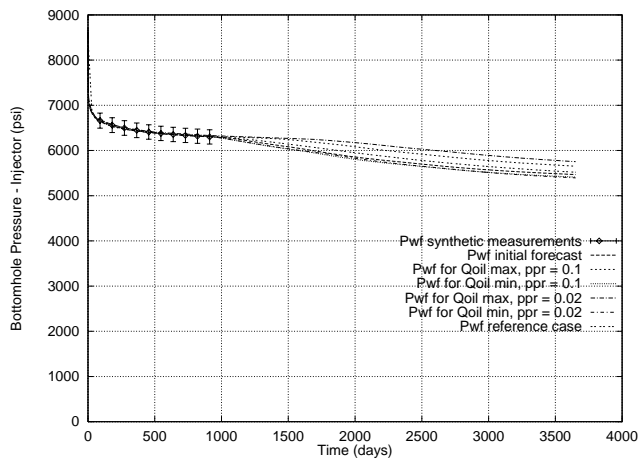


Fig. 16: Bottomhole pressure forecasts

A significant asymmetry may be observed between the minimum forecast and the maximum forecast in comparison with the forecast of the optimal model. The minimum production forecast is in fact rapidly bounded by the last measurement points of the historical data, which set a lower limit to the time of gas breakthrough in the

producing well. On the contrary, the maximum forecast corresponds to a gas breakthrough time delayed to the extreme.

In a second phase, the procedure for finding extreme models can be repeated for different probability criteria. The forecasts obtained for  $ppr = 0.02$  are shown from Fig. 14 to 16. The max. forecast for the oil flow rate is increased, when the min. forecast is limited by the breakthrough time. Figure 17 shows the extreme forecast envelope for a probability ratio varying from 0.02 to 1. In this way it may be observed that the uncertainty margin on the production forecasts does not increase significantly for models with probability lower than 0.05.

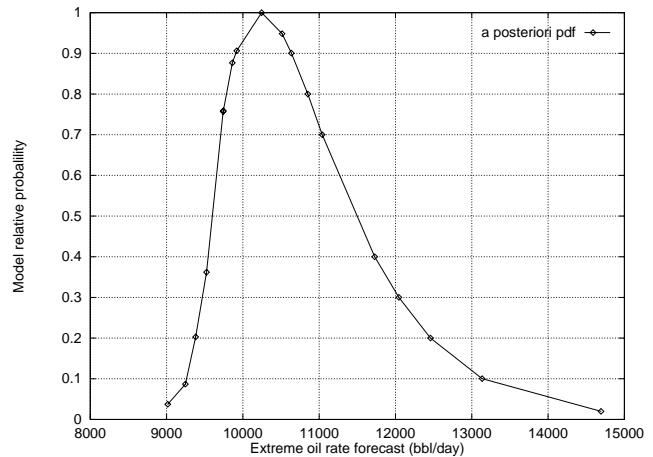


Fig. 17: a posteriori pdf associated with the extreme oil rate forecasts

This study shows that the probability distribution of a given production forecast criterion can be evaluated while considerably economizing CPU time on the model. A stochastic approach would in fact require a very large number of numerical simulations to scan the uncertainty domain with seven parameters or more.

## CONCLUSION

The Bayesian approach serves to understand the problem of history matching by considering the simulation model in a probabilistic form. The integration of all the available data, including the historical data and the initial geological knowledge, enables reduction of the uncertainties and improved characterization of the reservoir.

The methods used to forecast production uncertainties are complementary to history matching methods. These methods are aimed at quantifying the impact of the uncertainties of the model parameters on the production forecasts. Two types of approach have been compared: the stochastic approach and the deterministic approach.

The stochastic approach enables characterization of the variance of the forecasts by scanning the uncertainty domain, based on a large number of model realizations. This approach is well adapted for the treatment of geostatistical models. The stochastic approach enables quantification of

the variance of the forecasts due to a random seed, or to characterize the average geostatistical parameters of a distribution such as mean values, standard deviations and correlation lengths.

The deterministic approach consists of using an optimization process to directly find the images of the model which correspond to extreme forecasts of a given production criterion. A statistical interpretation is possible, by sourcing the extreme images in a confidence domain, corresponding to a probability criterion. The deterministic approach offers the advantage of considerably reducing the cost in terms of the number of simulations. The drawback is that it is more difficult to generalize for processing geostatistical images.

The proposed optimization algorithms integrate the problem of history matching and of uncertainty forecasting in a global approach. Tools such as the gradient method facilitate the analysis of the results of a simulation and simplify sensitivity studies to the parameters.

The proposed algorithms have been evaluated on a synthetic case. The possibility of integrating measurements of different types was demonstrated, for the simultaneous characterization of several parameters. The deterministic approach makes it possible to consider the procedures of matching and forecasting uncertainties as automatic or semi-automatic, with the possibility that the reservoir engineer can intervene. Feasibility on real case studies is still to be demonstrated.

Prospects are high for the use of this procedure, the goal being that the reservoir engineer has a set of tools and methods for a more effective reservoir study. To achieve this, a research and validation effort will be undertaken, to offer greater flexibility, particularly in the choice of parameters and the possibilities to account for a priori knowledge.

## **ACKNOWLEDGMENTS**

The authors thank the "Institut Français du Pétrole" (IFP) for their permission to publish this work.

This research has been partially funded by the European Community (Joule 2 topic 5 program). The modified SPE1 test case has been defined jointly by the participants of this project (TNO, IC, DTH/TU, NCC, ELF, IFP).

ATHOS® is a fluid flow simulator owned jointly by the "Institut Français du Pétrole" and BEICIP-FRANLAB.

## **REFERENCES**

- Jacquard P., "Théorie de l'interprétation des mesures de pression", revue de l'IFP, 1961, pp. 297-334.
- Jacquard P., Jain C., "Permeability Distribution from Field Pressure Data", SPEJ, Dec. 1965, pp. 281-294.
- Chavent G., Dupuy M., Lecomnier P., "History Matching by use of Optimal Control Theory", SPEJ, pp. 74-86, Feb. 1975.
- Chardaïre C., Chavent G., Jaffré J., Liu D., Bourbiaux B., "Simultaneous Estimation of Relative Permeability and Capillary Pressure", SPE 19680, 64th fall meeting, Oct. 8-11, San Antonio, Texas, 1989.
- Blanc G., Guérillot D., Rahon D., Roggero F. and Helios Reservoir Group, "Building Geostatistical Models Constrained by Dynamic Data — A Posteriori Constraints", SPE/NPF European 3D Reservoir Modelling Conference, Stavanger, April 16-17, 1996.
- Montadert L., Guérillot D., "Uncertainties for Production Forecasts During the Appraisal Phase and the Beginning of Production", Advances in Reservoir Technology, IBC conference, London, Nov. 1994.
- Bos C.F.M., van Kruijsdijk C.P.J.W., "How to Improve Risk Management Through Geosciences?", paper presented at the OAPEC/John Brown workshop on New Technologies Applied to Hydrocarbon Production, Delft, The Netherlands, Sept. 12-15, 1995.
- Bos C.F.M., Floris F.J.T., "Flow Constrained Reservoir Characterization by Bayesian Inversion of Reservoir Simulation Models", paper K-53 presented at the European Association of Petroleum Geoscientists & Engineers Conference, Vienna, Austria, June 6-10, 1994.
- Floris F.J.T., Bos C.F.M., "Flow Constrained Reservoir Characterization using Bayesian Inversion", 4th European Conference on the Mathematics of Oil Recovery, RØros, Norway, June 7-10, 1994.
- Tarantola A., "Inverse Problem Theory, Methods for Data Fitting and Model Parameter Estimation", Elsevier Science Publishers, Amsterdam, 1987.
- Guérillot D., Roggero F., "Matching the Future for the Evaluation of Extreme Reservoir Development Scenarios", 8th European Symposium on Improved Oil Recovery, Vienna, Austria, May 15-17, 1995.
- Anterion F., "History matching: a one day long competition: classical approaches versus gradient method", first international forum on reservoir simulation, Alpbach, Austria, Sept. 12-16, 1988.
- Anterion F., Eymard R., Karcher B., "Use of Parameter Gradients for Reservoir History Matching", SPE18433, Houston, TX, Feb. 6-8, 1989.
- Rahon D., Blanc G., Guérillot D. and Helios Reservoir Group, "Gradient Method Constrained by Geological Bodies for History Matching", 5th European Conference on the Mathematics of Oil Recovery, Mining University leoben, Austria, Sept. 3-6, 1996.
- Bissell R.C., Sharma Y., Killough J.E., "History Matching Using the Method of Gradients: Two Case Studies", SPE 28590, Houston, Sept. 25-28, 1994.
- Bissell R.C., "Calculating Optimal Parameters for History Matching", 4th European Conference on the Mathematics of Oil Recovery, RØros, Norway, June 7-10, 1994.
- Marquardt D.W., "An Algorithm for Least Squares Estimation of non-linear Parameters", J. Soc. Indust. Appl. Math., pp. 431-441, 1963.
- Powell M.J.D., "A Hybrid Method for non-linear Equations", in "Numerical Methods for non-linear Algebraic Equations", ed. P. Rabinowitz, Gordon&Breach, p. 87, 1970.
- Fletcher R., "Practical Methods of Optimization", John Wiley & Sons, Inc., New York, 1987.